Topology of Surfaces

Most of this material is from *The Shape of Space* by Jeff Weeks. The word searches were taken from Torus Games at http://geometrygames.org/TorusGames/index.html . You can download the Torus Games app for many other games.

1 Geometry vs. Topology

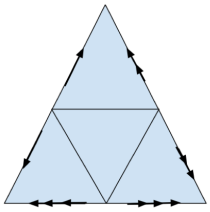
- The properties of an object that change when you bend or stretch it are the geometry of the object.
  - For example, angles and distances are parts of geometry but not topology.
- The properties of an object that stay the same when you bend or stretch it are called the topology of the object.
  - Two objects are considered the same topologically if you can deform one into the other without tearing, cutting, fusing, or other violent actions.

"Mug and Torus morph". Licensed under Public Domain via Wikimedia Commons

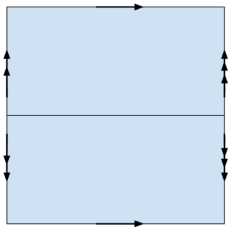
1. Which surfaces are topologically the same?

2 Gluing Diagrams

2. What topological surface do you get when you glue (or tape) the edges of the triangle together as shown?

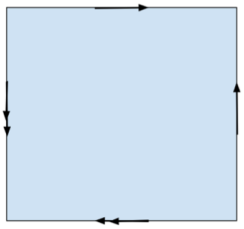


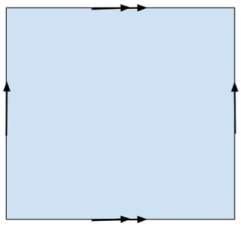
3. What do you get when you glue the edges of the square together like this?



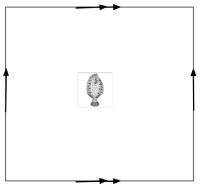
Don't glue the interior parts of the square together, just the edges!

4. What surface is this? And this?





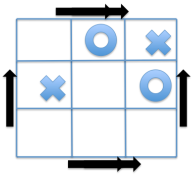
5. What happens as this 2-dimensional creature travels through its tiny universe?



What does it see when it looks forwards? Backwards? Left? Right? (See Torus Games animation.)

3 Tic-Tac-Toe on a Torus

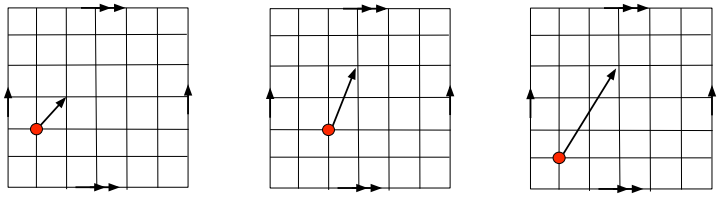
6. Where should X go to win? What if it's O's turn?



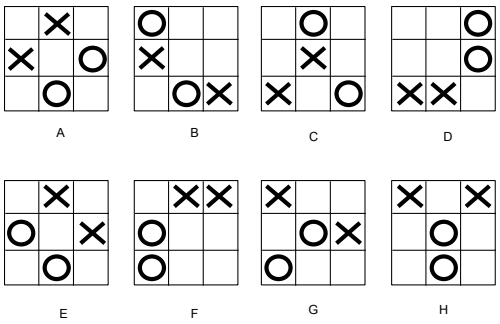
7. Play some games of Tic-Tac-Toe on the Torus with a classmate.

4 Torus Problems

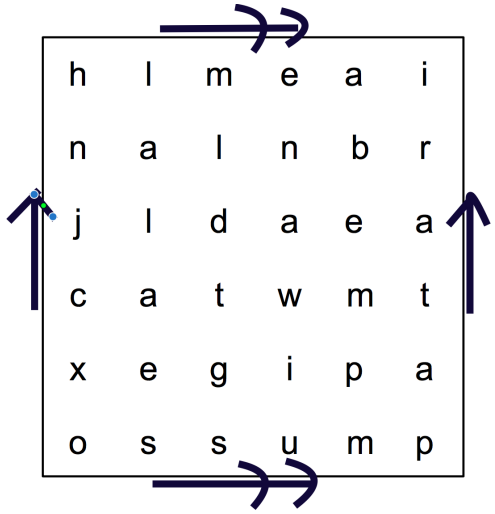
8. A ladybug on a torus walks in a straight line until she returns to her starting point. Draw her path and find the length of her path if:
   (a) she walks 1 unit northward for every 1 unit eastward, as shown at the left.
   (b) she walks 2 units northward for every 1 unit eastward as shown in the center.
   (c) she walks 3 units northward for every 2 units eastward as shown at right.



9. Which of the following positions are equivalent in torus tic-tac-toe?



10. How many essentially different first moves are there in torus tic-tac-toe?
11. Is there a winning strategy for the first player in Tic Tac Toe on the torus? That is, is it possible for the first player to win no matter what the second player does?
12. A Cat's Game in Tic Tac Toe is a game where neither side wins, even though the board is filled up with X's and O's. Is it possible to have a Cat's Game in Tic Tac Toe on the torus?
13. Try this word search on the torus. Remember that the top and the bottom are glued together and the left and right sides are glued together.



bat
camel
cat
jaguar
llama
ox
panda
pig
possum
puma
rat
seal
shrew
tapir