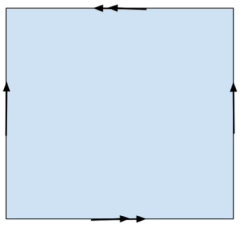
The Kelin Bottle

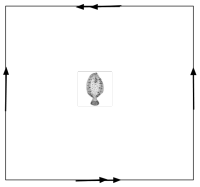
Most of this material is from *The Shape of Space* by Jeff Weeks. The word searches were taken from Torus Games at http://geometrygames.org/TorusGames/index.html . You can download the Torus Games app for many other games.

1 The Klein Bottle

1. What surface do you get when you glue together the sides of the square as shown?



2. What happens as this creature travels through its Klein bottle universe?

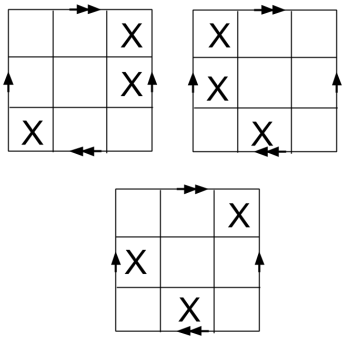


   - A path that brings a traveler back to his starting point mirror-reversed is called an orientation-reversing path.
   - A surface that contains an orientation-reversing path is called non-orientable.

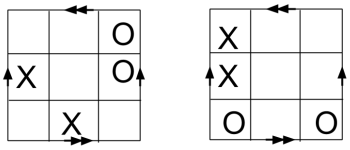
3. Can you find more than one orientation reversing path in this surface?

2 Tic-Tac-Toe on a Klein Bottle

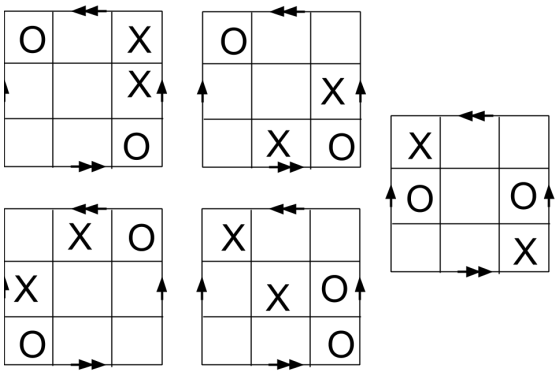
4. Which of these are winning positions in Klein bottle Tic-Tac-Toe?



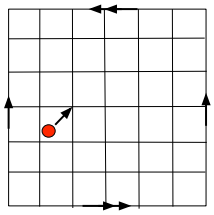
5. Where can X go to win immediately in Klein bottle Tic Tac Toe?



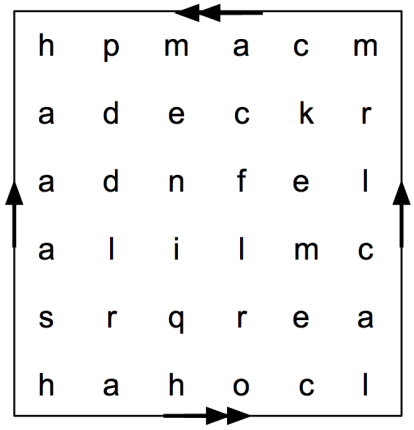
6. Play a few rounds of Klein bottle Tic-Tac-Toe with a classmate.
   - Is there a winning strategy?
   - Is it possible to get a cat's game?
   - How many essentially different first moves are there?
7. What are the best moves for X in these positions?



8. A ladybug on a Klein bottle walks in a straight line until she returns to her starting point. She walks 1 unit northward for every 1 unit eastward. Draw her path.

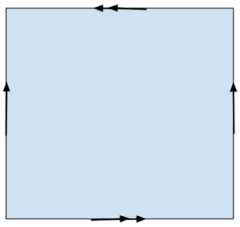


9. Try this word search on the Klein bottle. The arrows show how the sides are glued together.



ash
birch
cedar
elm
fir
larch
lilac
maple
oak
palm
pine
poplar

10. What happens when you cut a Klein bottle in half? Hint: It depends on how you cut it.



Cutting a Klein bottle in half - animation

Cutting a Klein bottle in half - IRL

3 3-manifolds

11. Is there a 3-dimensional analog to the torus? What about the Klein bottle?