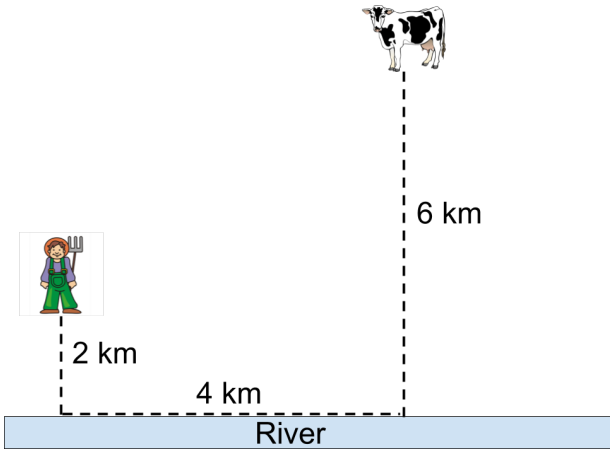
Reflection Does It! Part 2

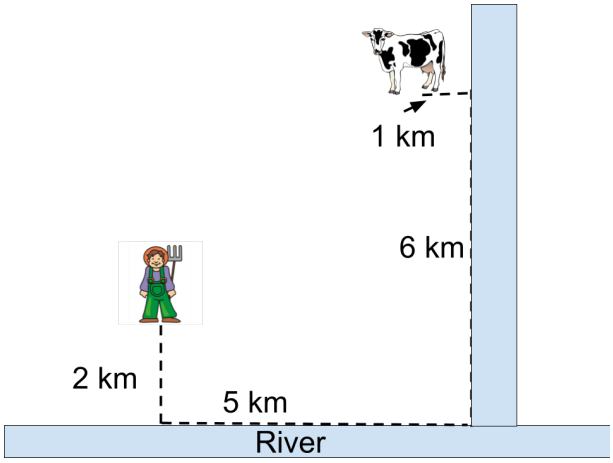
February 20, 2021

Farmer and Cow

1. During a hot summer day, a farmer and a cow find themselves on the same side of a river. The farmer is 2 km from the river and the cow is 6 km from the river. If each of them would walk straight to the river, they would find themselves 4 km from each other. Unfortunately, the cow has broken its leg and cannot walk. The farmer needs to get to the river, dip his bucket there, and take the water to the cow. To which point on the river should the farmer walk so that his total walk to the river and then to the cow is as short as possible? Prove your claim.



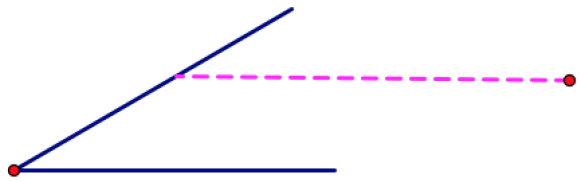
2. What if the farmer and the cow are near the fork of a river, like this one? Which branch of the river should the farmer go to, and where on the river should he go?



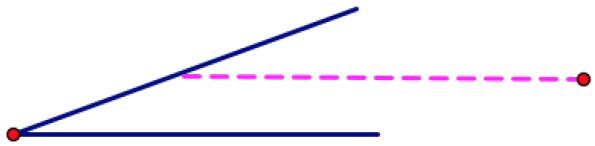
Hinged Mirrors

Two hinged mirrors form a 30° angle. A light beam enters the hinged mirror angle parallel to one of the sides and is reflected from the sides according to the usual law that the angle of incidence is equal to the angle of reflection.

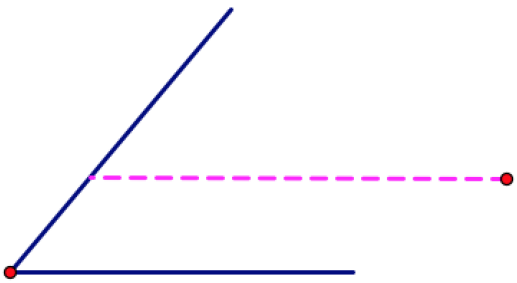
3. Prove that the beam will eventually leave the hinged mirror angle. How many times will it reflect off of the mirrors before leaving?



4. What if the angle between the mirrors is 20 degrees?

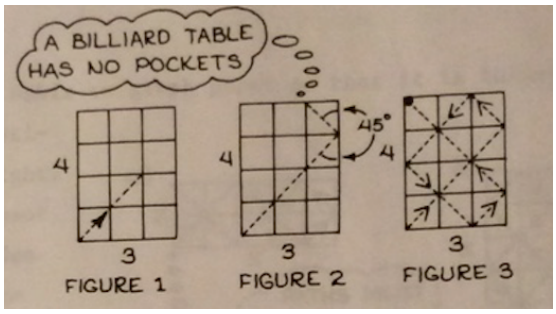


5. What if the angle is 50 degrees?

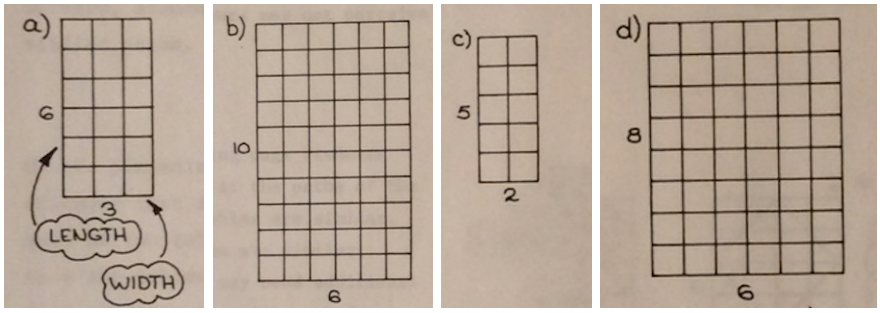


Pool Tables

6. The figures below represent the same billiard table. A ball has been hit from the lower left hand corner so that it travels at a 45° angle from the sides of the table (Figure 1). When the ball strikes the cushion of the table, it rebounds from it at the same angle (Figure 2). If the ball continues, it will reach the upper left hand corner (Figure 3).

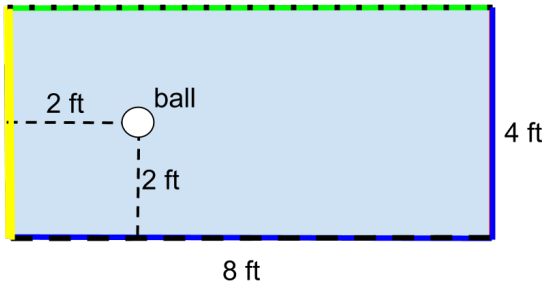


For each table below, draw the path of the ball until it strikes a corner. Always shoot the ball along a 45° path from the lower left hand corner.



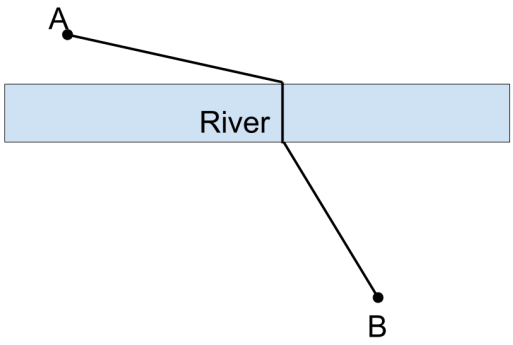
7. A ball is shot from the corner of a pool table at a 45 degree angle. Which corner of the pool table will teh ball reach first, and how many times will it hit the sides until it does so, if (a) the pool table is 3×5, and (b) the table is 3×1001?

8. A pool table is 4 feet wide and 8 feet long, as drawn below. You strike a ball that is initially placed 2 feet over and 2 feet up from the bottom left hand corner, so that its first bounce is against the dotted wall. In how many places can you hit that dotted wall so that the ball goes into the pocket on the bottom right corner after exactly 3 total bounces off the dotted and dashed walls? It doesn't matter how many times the ball bounces off the walls drawn with solid lines.

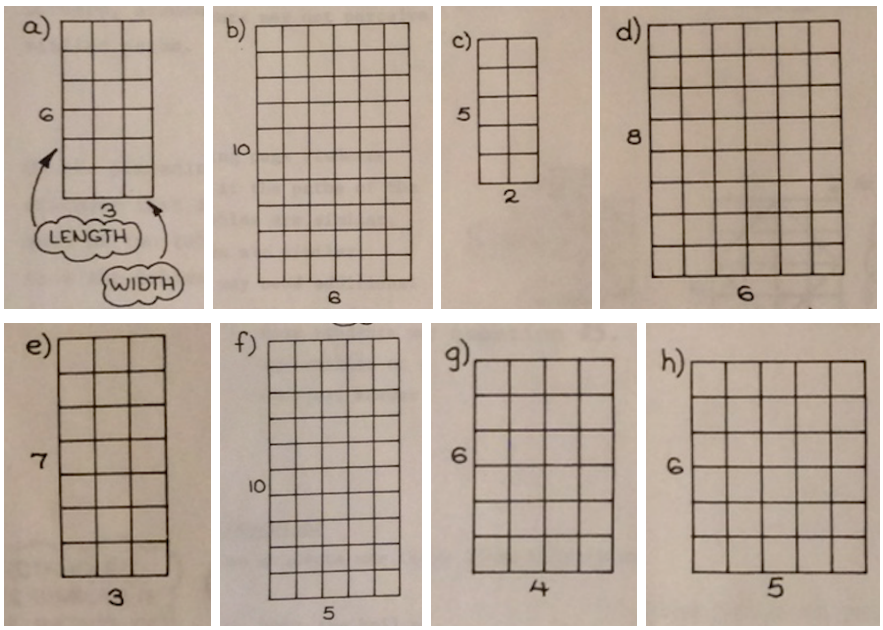


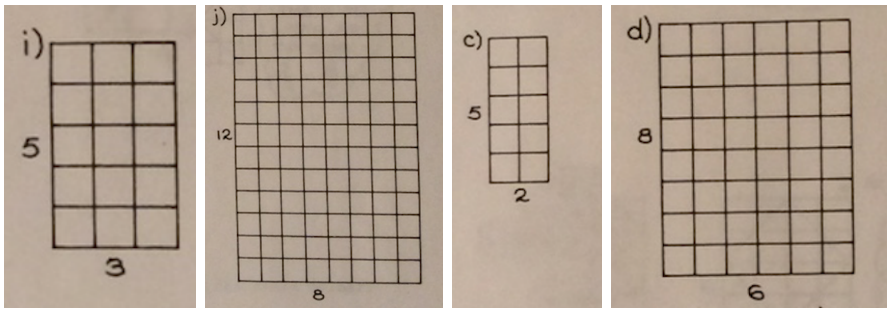
Extras

9. Two towns, A and B, are on opposite sides of a canal with parallel straight sides. A road with a bridge that crosses the canal perpendicularly is to be built. Where should the bridge be located so that the length of the road from A to B is to be minimized?



10. For each table below, draw the path of the ball until it strikes a corner. Always shoot the ball along a 45° path from the lower left hand corner.





11. What do you notice about the paths on Tables (g) and (j)?

12. Name another pair of tables that have similar paths.

13. Table (c) has a width of 2, a length of 5. Predict the dimensions of a table on which a ball will travel a similar path: width length . Test your prediction by drawing the table and the path on graph paper.

14. What will the paths on a table with width 6 and length 10 look like? A table with width 15 and length 20?