Geometry for Middle School Math Competitions

- *Chapel Hill Math Circle*

In this warm-up activity, we will explore a few of the most famous and most frequently used geometry patterns that you can use in math competitions. Then we'll try some MathCounts Geometry problems.

1. Use the Pythagorean Theorem to find the length of the diagonal of a square with side length of 10. Express your answer in simplest radical form. *(Not sure about "Pythagorean Theorem" or "Simplest Radical Form"? - ask the person beside you or a volunteer.)* Repeat the process for a square with side lengths of 7 units.

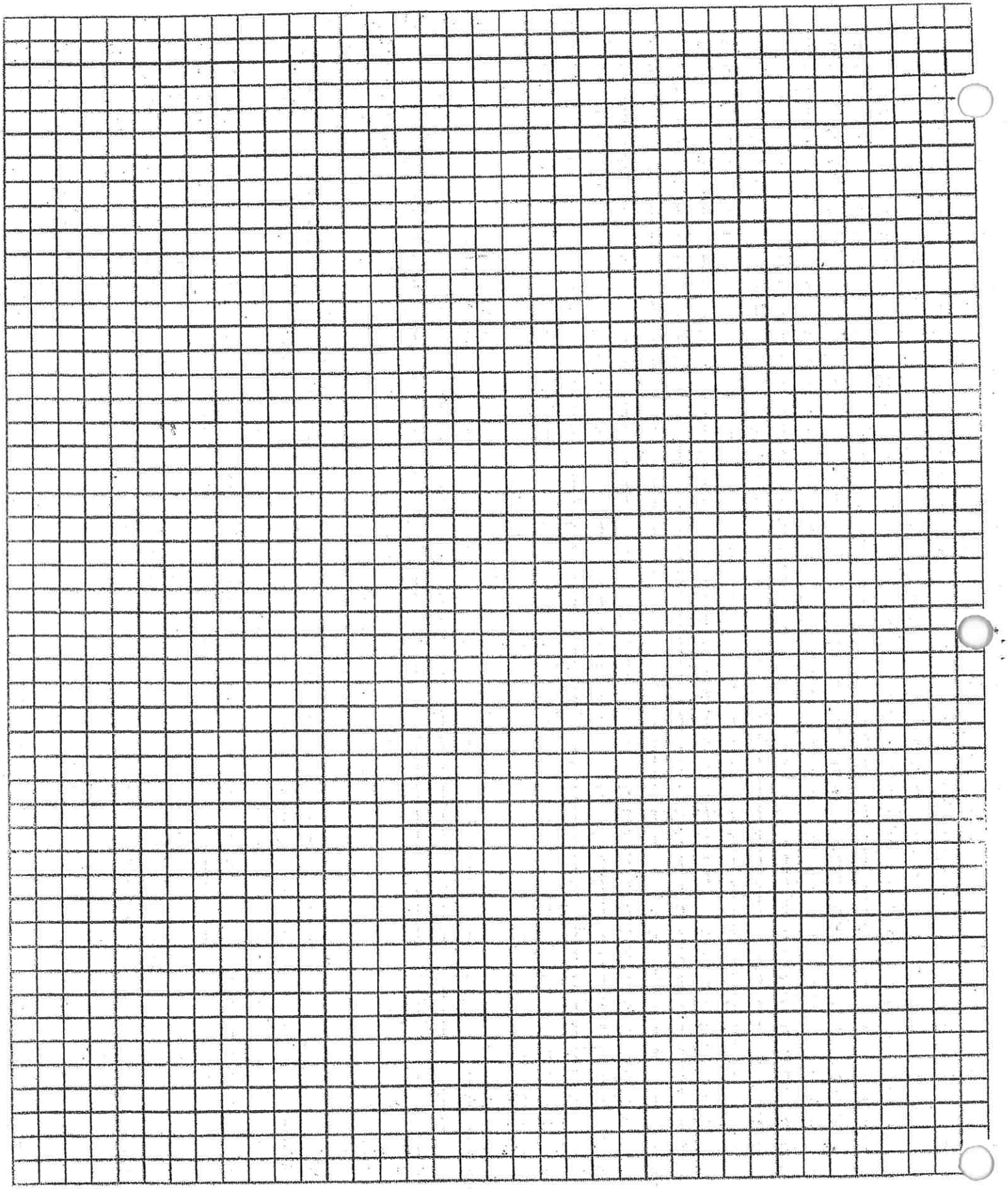
2. Use the Pythagorean Theorem to find the height of an equilateral triangle with side lengths of 10. Express your answer in simplest radical form. Repeat the process if the side length was 7.

3. The most common and famous "Pythagorean Triple" is the 3-4-5 right triangle. These three sides make a right triangle because $3^2 + 4^2 = 5^2$ $(9 + 16 = 25)$. Most right triangles have at least one side that is an irrational number such as the 5-6-$\sqrt{61}$right triangle. We call it a Pythagorean Triple when all three numbers are nice, clean natural numbers. List other Pythagorean Triples that you know of or can discover.

4. The three interior angles of a triangle (n=3) sum to 180°. The four interior angles of a quadrilateral (n=4) sum to 360°. If this is the beginning of a pattern, what would the sum be for pentagons (n=5) or hexagons (n=6)? Can you state the pattern as a rule in terms of n?

5. If the polygon is "regular" (equilateral and equiangular), we can turn the rule above into a rule for the measure of each angle of a regular polygon with n sides. What is that rule? What would be the measure of each interior angle of a regular pentagon? A regular hexagon?

6. Regular hexagons are the most famous regular polygons because the diagonals split the inside into 6 identical equilateral triangles. Sketch a regular hexagon below, including its diagonals. What would be the area of this polygon if the side length were 10 units?



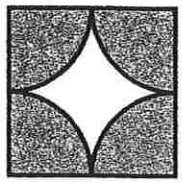
End-of-Course Graph Paper

2007-08 Math Counts Geometry (1)

LEAP Math w/ Dr. Blackburn

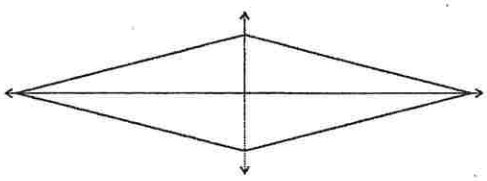
Name _____

u4 2. _____ sq cm Four congruent quarter-circles are drawn inside a square of side length 4 centimeters, as shown. What is the area of the shaded portion of the square region? Express your answer in terms of π.



u8 2. _____ sq units What is the area of the triangle with vertices at (0, 2), (3, 2) and (3, 0) on the coordinate plane?

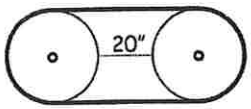
u10 7. _____ sq units



What is the area enclosed on the coordinate plane by the graph of the equation $|x| + |4y| = 20$?

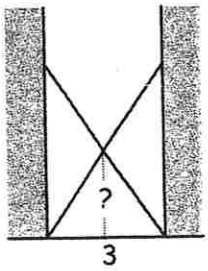
u10 2. _____ cu in A rectangular block has faces with areas of 48 in^2, 72 in^2 and 96 in^2. What is the volume of the block?

o4 8. _____ inches



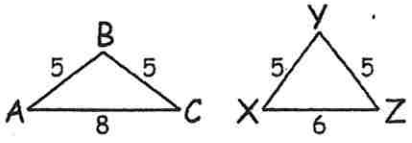
Each circular pulley shown has a radius of 12 inches. The shortest distance between the pulleys is 20 inches. What is the length of the continuous belt that loops around both pulleys? Express your answer to the nearest whole number.

o4 9. _____ meters Two ladders, both 6 meters in length, are leaned up against opposite vertical walls in a 3-meter-wide corridor, as shown. How far above the ground do the two ladders cross? Express your answer as a decimal to the nearest tenth.

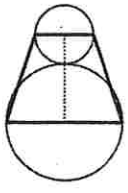


u4 9. _____ Several points are plotted on a graph. For each point, the x-coordinate is the length of a side of a square while the y-coordinate is the perimeter of that same square. One such point is (2, 8) since a square with side length 2 units has a perimeter of 8 units. What is the slope of the line connecting the points? Express your answer in simplest form.

u4 10. _____ sq units Triangle ABC has side lengths 5, 5 and 8 units. Triangle XYZ has side lengths 5, 5 and 6 units. What is the difference of the areas of these two triangles?



u9 4. _____ sq units



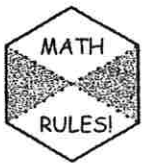
Two non-congruent circles are externally tangent to each other. Each base of an isosceles trapezoid is a diameter of one of the circles. If the distance between the centers of the circles is 9 units, what is the area of the trapezoid?

2007-08 Math Counts Geometry (2)

LEAP Math w/ Dr. Blackburn

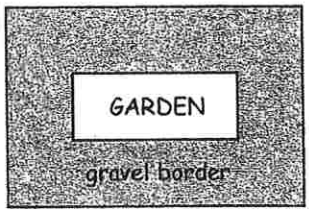
Name _____

u8 7. _____ sq inches The area of this sign in the shape of a regular hexagon is $96\sqrt{3}$ square inches. What is the total area of the two shaded regions? Express your answer in simplest radical form.

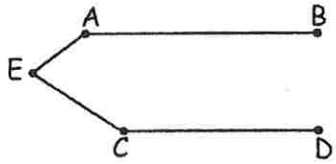


u8 8. _____ sq units Two sides of a particular isosceles triangle are 6 and 13 units. What is the area of this triangle? Express your answer in simplest radical form.

u8 9. _____ feet A rectangular garden is 10 feet by 4 feet. A gravel border with uniform width along the sides and 90° corners surrounds the garden, as shown. The area of the gravel border is six times the area of the garden. What is the perimeter of the outside of the gravel border?

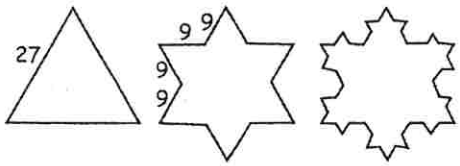


u8 10. _____ degrees In the figure shown, segments AB and CD are parallel. What is the sum of the measures of angles BAE, AEC and ECD?



u9 2. _____ degrees How many degrees are there in the smallest angle between the two hands of a clock at 2:30?

u7 7. _____ units In the sequence of equilateral figures shown, the middle third of each segment is replaced with two segments that are each the same length as the replaced piece. Each side of the first figure (the triangle) is 27 units. What is the perimeter of the third figure in the sequence?



u7 3. _____ sq units What is the area of regular hexagon ABCDEF if AB = 2 units? Express your answer in simplest radical form.

05 8. _____ sq ft A 20-foot-high rectangular room has a floor that measures 18' by 15'. Its doorway measures 3' by 12', and its only window measures 7' by 10'. How many square feet of wall space does the room have?

05 2. _____ sq units An acute angle of a right triangle is 30°, and the hypotenuse is 40 units. What is the area of the triangle? Express your answer as a decimal to the nearest tenth.

03 1. _____ feet The formula for the total surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. A particular solid right cylinder of radius 2 feet has a total surface area of 12π square feet. What is the height of this cylinder?