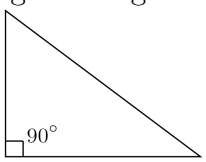
Pythagorean Theorem

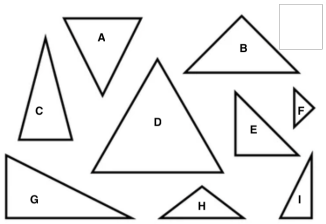
Definitions

A right triangle is a triangle that has one "right angle" – angle that is 90° like the corner of a book.



The longest side of a right triangle, that is opposite the right angle, is called the *hypotenuse*.

1. Which of these triangles are right triangles?



Side Length of right triangles

2. Use a ruler to draw two triangles on your paper: one triangle that is a right triangle and one that is not. You can use the corner of the ruler or the corner of a piece of paper to make a right angle.

3. Measure the lengths of the sides of the two triangles in centimeters and write them below and on the board.

Right Triangle?	shortest side (a)	middle side (b)	longest side (c)	$a+b$

4. How does the sum of the lengths of the two shorter sides compare to the length the longest side? That is, how does $a+b$ compare to c? Is $a+b$ bigger, smaller, or the about the same as c?

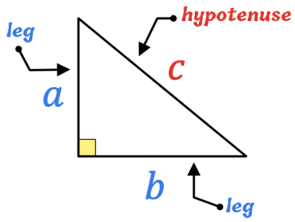
5. Make a chart of the squares of the lengths of the sides along with the sum of two smaller squares..

Right $\triangle$?	short side (a)	mid side (b)	long side (c)	a^2	b^2	c^2	a^2+b^2

6. How does a^2+b^2 compare to c^2? Is a^2+b^2 bigger, smaller, or the about the same as c^2?

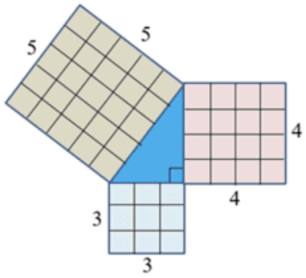
Pythagorean Theorem

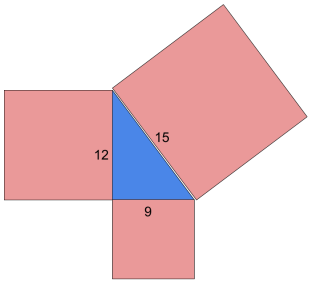
In fact, for any right triangle, $a^2 + b^2$ should equal c^2, if you measure carefully, where a and b are the lengths of the two shorter sides and c is the length of the longest side, the hypotenuse.

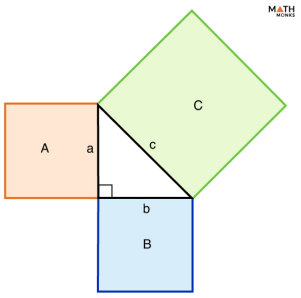


$$a^2 + b^2 = c^2$$

7. In the pictures, below, what are the areas of the squares?



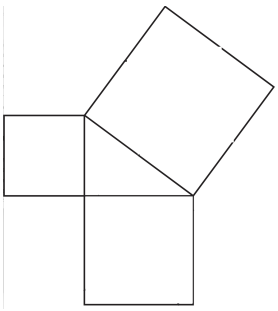


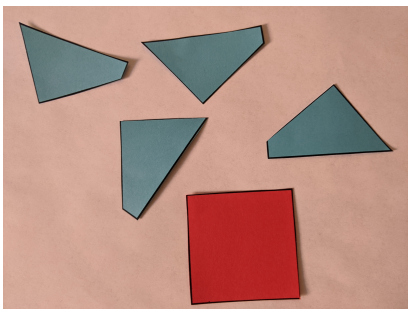


8. What does the Pythagorean Theorem say about the areas of the squares that rest on the sides of a right triangle?

Perigal's Puzzle

9. Use the pieces to fill the a square and the b square (the two smallest squares). Then use the same pieces to fill the c square (the largest square).

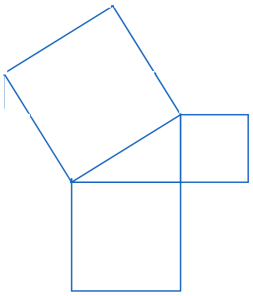


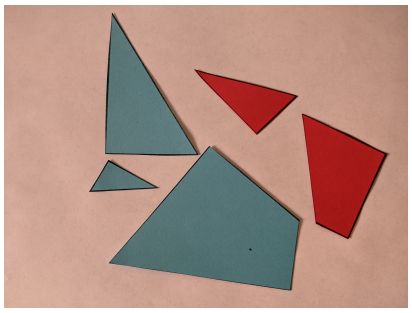


10. What does this say about the three numbers a^2, b^2, and c^2?

Qurra's Puzzle

11. Use the pieces to fill the a square and the b square (the two smallest squares). Then use the same pieces to fill the c square (the largest square).

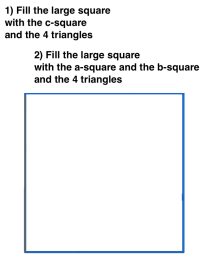


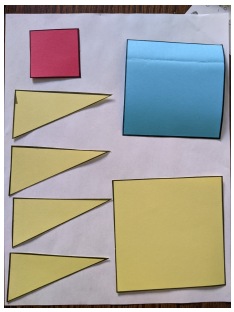


12. What does this say about the three numbers a^2, b^2, and c^2?

Bhaskara's Puzzle / Zhou's Puzzle

13. Use the a square plus the b square plus the 4 triangles to fill in the large square. Then use the c square plus the 4 triangles to fill in the large square.

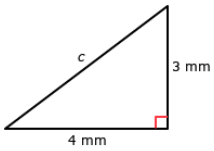


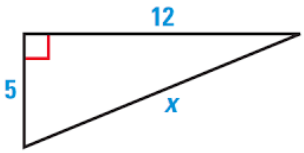


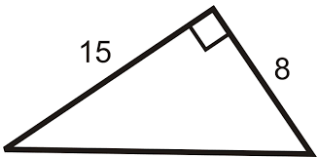
14. How does this prove that $a^2 + b^2 = c^2$?

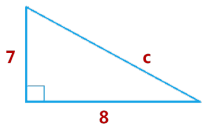
Exercises

15. Find the missing side length.



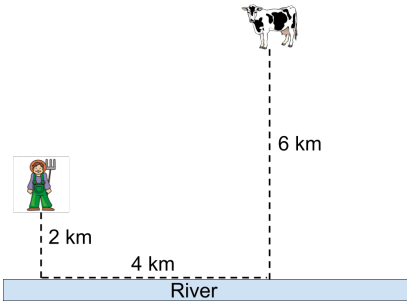




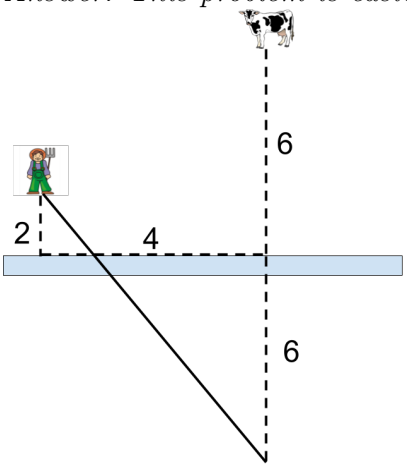


The Farmer and the Cow

16. During a hot summer day, a farmer and a cow find themselves on the same side of a river. The farmer is 2 km from the river and the cow is 6 km from the river. If each of them would walk straight to the river, they would find themselves 4 km from each other. Unfortunately, the cow has broken its leg and cannot walk. The farmer needs to get to the river, dip his bucket there, and take the water to the cow. To which point on the river should the farmer walk so that his total walk to the river and then to the cow is as short as possible? Prove your claim.



Answer: This problem is easiest to solve by reflecting the cow over the mirror line of the river.



Then the shortest distance between the farmer and the mirror image cow will just be a straight line. The two triangles are similar, and the ratio of their vertical sides is 6 to 2, or 3 to 1. Therefore, the ratio of their horizontal sides should also be 3 to 1. So the point of the river should divide the distance 4 along the river into a 3 to 1 ratio, in other words, the farmer should walk to a point on the river that is 1 km east of where he is now.