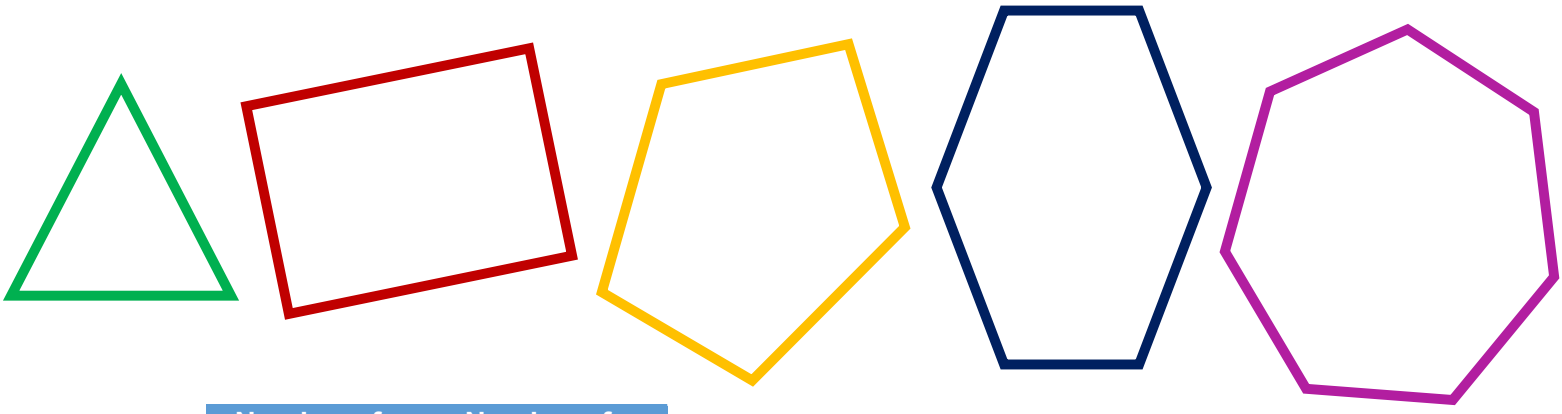
Chapel Hill Math Circle
Session 5 – November 18, 2023: Patterns in Diagonals; Angles
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com
Supplies needed: rulers, protractors, paper, colored pencils

Welcome to Chapel Hill Math Circle! We're glad that you are here to have some fun! Let's finish talking about diagonals for now (and review what we learned first if you didn't make the last session) and move on to angles.

Diagonals

We decided that a diagonal is a straight line segment between two "vertices" or corners of a polygon that aren't directly connected. Many of you made a table of diagonals. Let's look at the numbers that many of you found and try to find a pattern.

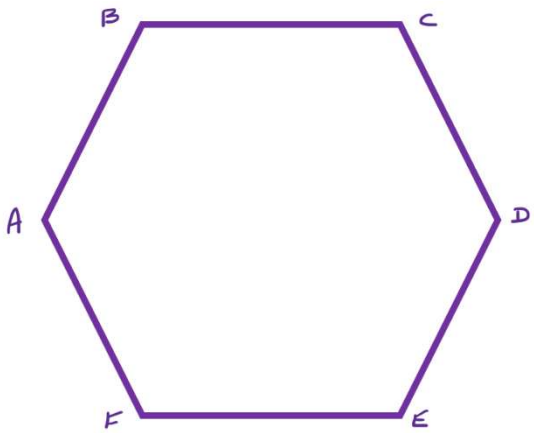


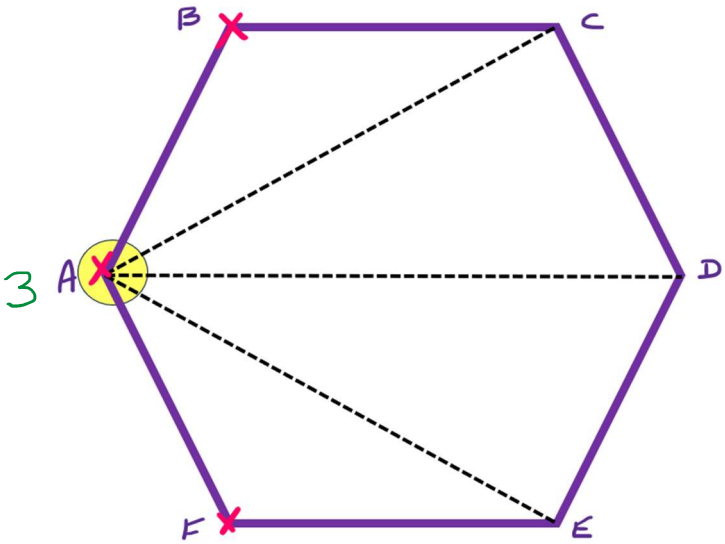
Number of sides	Number of diagonals
3	0
4	2
5	5
6	9
7	14
8	20
9	27
10	35
n	

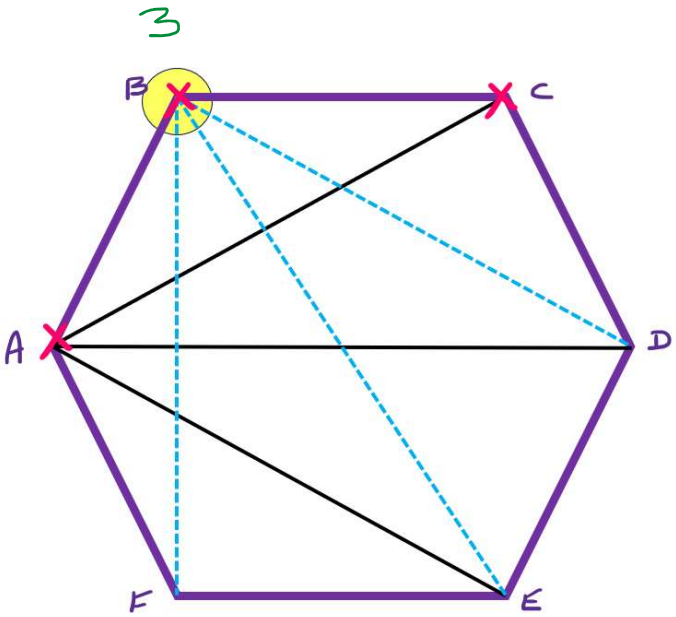
How we counted diagonals

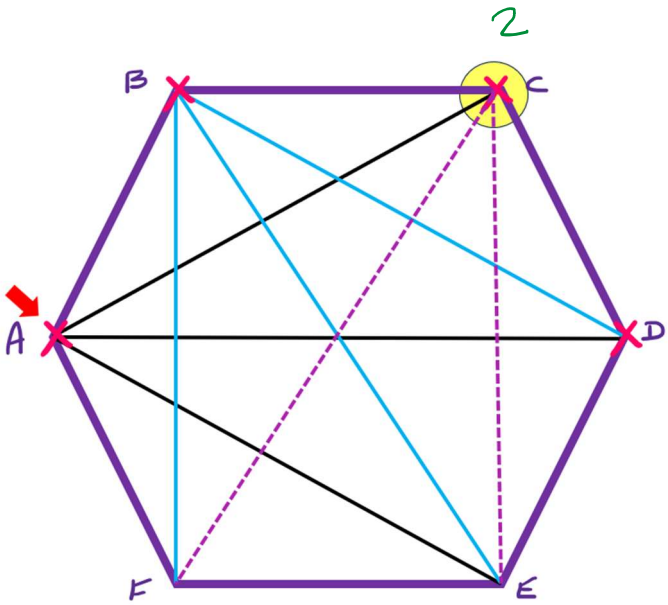
- What patterns do you see in the table?
- Can we predict the number of diagonals for any number of sides?

Consider the 3 pictures on the next page for a hexagon (6-gon). Do they help us find a pattern? How many (new) diagonals will we have from D? E? F? Let's discuss in a few minutes!









Your notes on patterns in counting diagonals

In our discussion we will talk about the “-3 -3 then countdown” rule. We’ll also discuss another approach of keeping our vertices happy and letting each vertex visit all but 3 of the vertices – and then fixing things so we don’t have double roads. Cool stuff!

Your new notes on patterns in counting diagonals

Angles

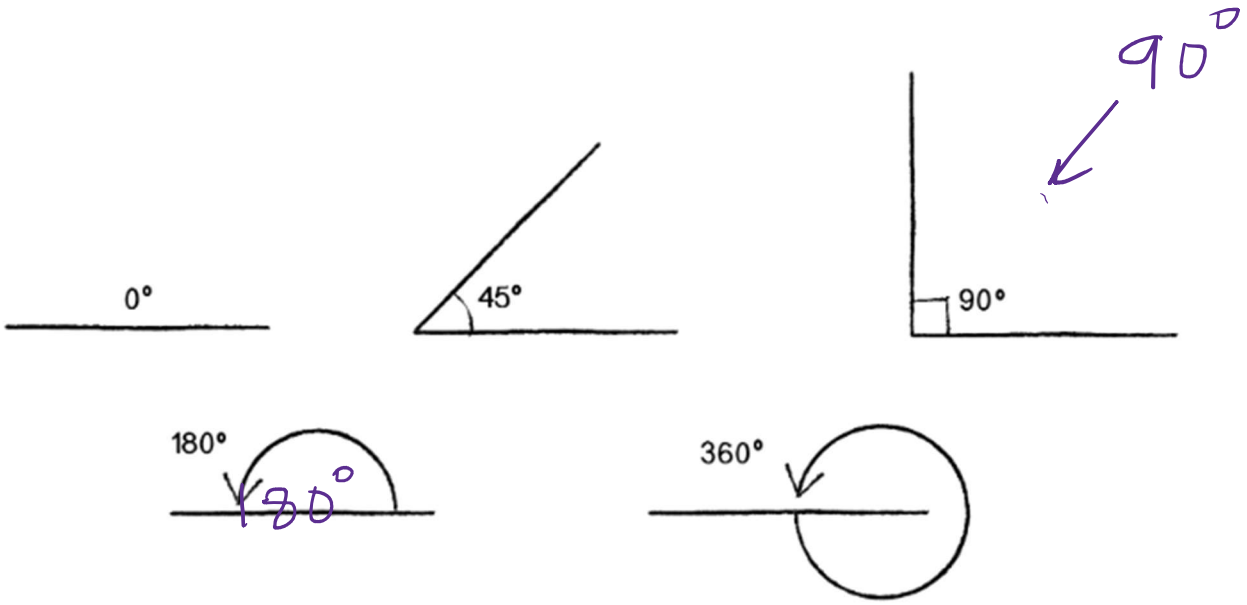
What is an angle? Take a minute to draw some pictures of angles. If you don’t know what an angle is, think about triangles having 3 angles.

Doodling about what an angle is

(Don't peek till I tell you!)

I can't wait to see your drawings and thoughts. Let's define an angle as the space formed inside two intersecting lines or line segments. We typically measure the space by a number called the degree that goes from 0 to 360. Khan Academy[1] has a nice summary image showing some angles.

I like to compare angles to the opening of a book. You might have heard of obtuse angles, acute angles, right isosceles triangles, equilateral triangles, and more. Don't worry about these names for now. And it's okay if you haven't heard of them!



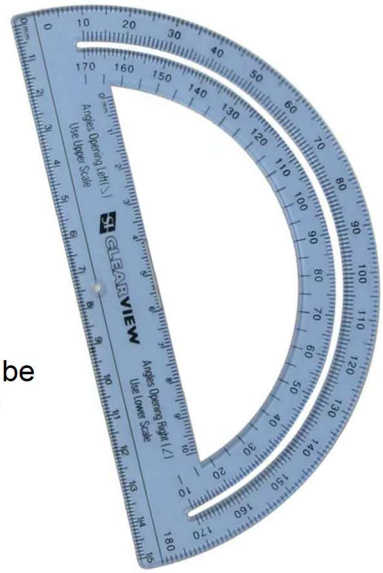
A right angle or 90 degree (90°) angle is like the corners of a piece of paper. Degrees numbered between 0 and 90 measure how open a book is. Take a few moments to estimate some angles. What might a 90° angle look like? How about a 45° one (note that 45+45 = 90)? 100°? 10°? Draw your angles and your estimates of their angle measurements here.

Doodling about what an angle is

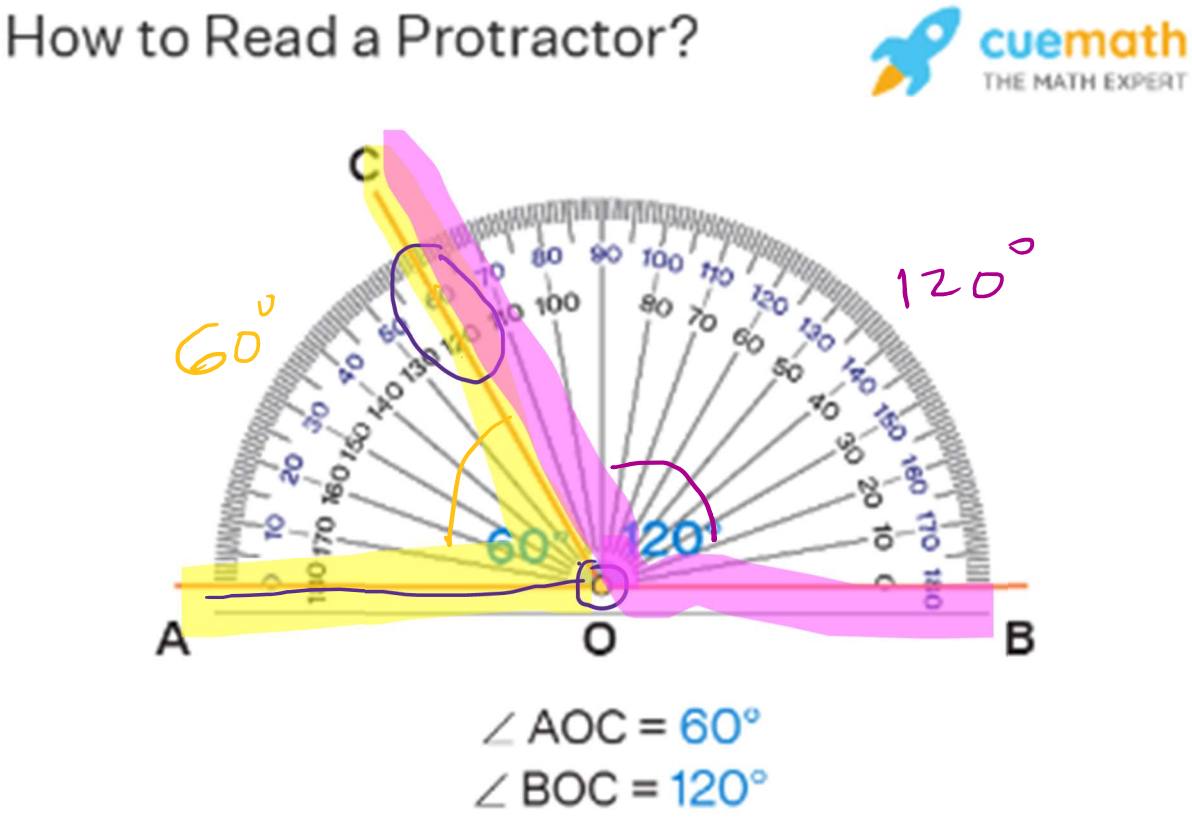
[1] khanacademy.org/computing/computer-programming/programming-natural-simulations/programming-angular-movement/a/angles-and-units, accessed Nov. 8, 2023

Protractors

Have you ever heard of a protractor[2]? I will show you how to use it to measure angles with the help of a picture from cuemath[3].



- Use a ruler to make an angle.
- Line up the center hole (marked O below) with the angle's vertex and have one side of the triangle go along the line at the bottom of the protractor (A to O or O to B). Look at what numbers the other line goes through. In the example below, line OC goes through both 60 and 120.
- Angles smaller than a right angle will be between 0 and 90 degrees. Therefore AOC, which is less than a right angle will be 60°. BOC, which is more than a right angle, will be the bigger number, 120°.
- Note that a straight angle is 180°.



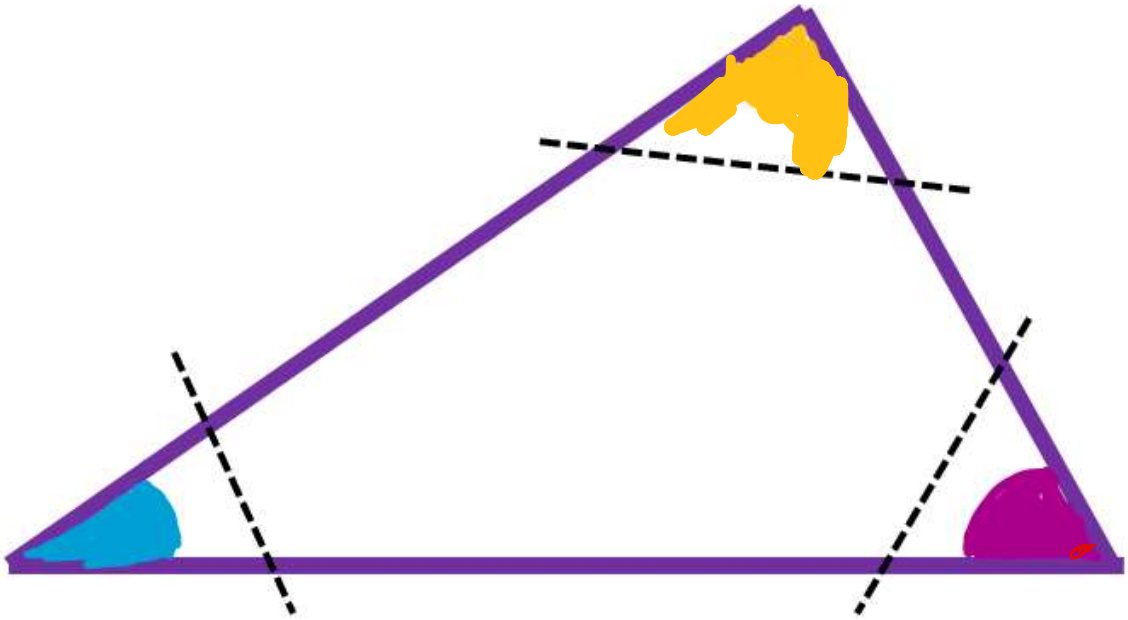
[2] The picture of the protractor is from Fisher Scientific, fishersci.com/shop/products/clearview-transparent-plastic-protractor/S90491A, accessed Nov. 8, 2023

[3] from cuemath.com/geometry/protractor, accessed Nov. 15, 2023

A straight angle is $180°$ … and the angles of a triangle sum to $180°$

As we saw on the previous page, a straight line can be viewed as a $180°$ angle. Make a few triangles. Make them different sizes, with different sizes of angles; try some that have corners like a page (a right angle or $90°$ angle); try some where all the angles are the same; experiment! In each of these, color the three angles different colors like I have below.

Then use scissors to cut out the three angles. You can even just rip; the cuts don't have to be clean. You should have something like the three pieces at the bottom. Shove them together and you will find that no matter what triangle you have started with, you have the three pieces forming a line. This shows that the angles of a triangle add up to a $180°$!



Keep on experimenting

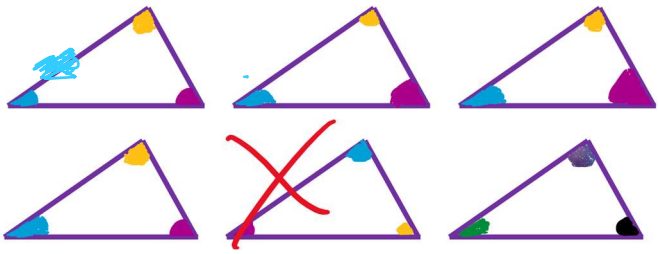
Play with your protractor. Make and measure different angles. What do you think 180° + 90° = 270° looks like? 180° + 180° = 360°?

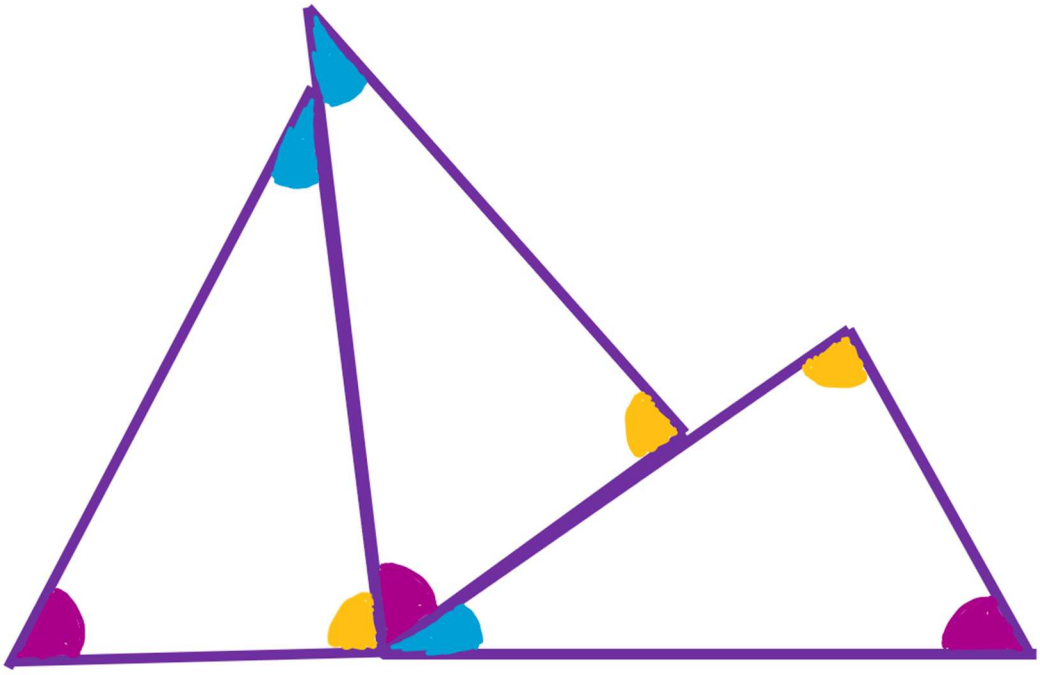
Here is a second way to show that the angles of a triangle add up to $180°$, or a straight line. Make three identical copies of a triangle. Here are a few ways to do this:

- Make one triangle then use tracing paper to make two more copies.
- Find a triangular object and trace around it to make three copies.
- With your family's help, find a printable triangle or have them help you make a printable triangle with a computer. Print 3 copies.

Once you have your three triangles, put them all in front of you so they are arranged the same way. Color each of the three angles in one triangle a different color just like before. Duplicate this for the other two; the amount of coloring doesn't have to be the same but the same angles in each triangle should have the same color.

It should look something like the top and not the bottom example here to the right.
Once you have done this, cut out the three triangles and arrange them so that each of the three colors meet at a point. Do the triangles come together as a line at the bottom? If so, how does this show that the angles of a triangle add up to $180°$?





Play with your protractor. Make and measure different angles. What do you think 180° + 90° = 270° looks like? 180° + 180° = 360°?

Have a good Thanksgiving!

If you have a protractor at home, keep on measuring. If you don't have a protractor, estimate. What fun angles can you find at home and when you're out and about?

Here are a few questions to think about.

The triangle is considered a strong shape. See if you can figure out why. Do you see triangles in roofs, bridges, playgrounds, and other places?

What happens when you add angles? Draw a 45° angle in one color. The pick another color and add on another 45° angle. What is 45+45? It is 40+50 = 90, so label this angle 90°. Continue on to 135°, 180°, and so forth. What do you notice happens after a while?

What do you think 360° looks like? How about a whole bunch of 360° turns like 10 of them (= 3600°)?

I hope you had fun! See you next time!

Have Fun!
Mr. Barman

P.S. Let your parents know that I host the country's largest vegetarian (all vegan) Thanksgiving! They can visit **trianglevegsociety.org** to learn more. Watch for us on the evening news on Thanksgiving!

Also, please let them that Mathematics Institute of the Triangle started a new 3rd grade class this past week; they can find out about our classes including 1st-3rd grade, middle school, prealgebra, algebra, geometry, and tutoring, at **trianglemathinstitute.com**