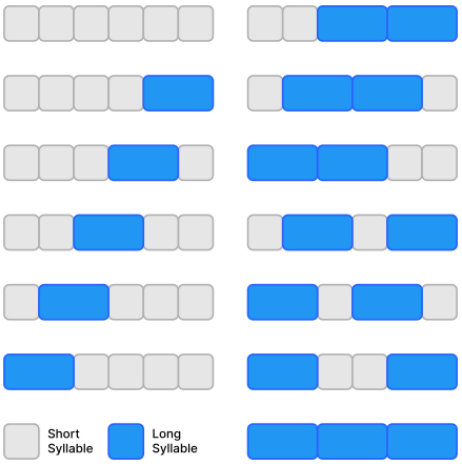
Fibonacci Day 11/23

PATHWAYS
#51 Geometry
#40 Sequences, Series, Summability
#68 Computer Science

Family Math Circle ~~~ November 2023

Pathways name our math inspirations using Mathematics Subject Classification. Mathematicians around the world label their work with library codes from MSC, https://msc2020.org/ Let us help students feel happy familiarity with each subject area!

Family Math Circle is an informal learning space where participants make advanced mathematics accessible to everyone in kind ways.



Thirteen (F_7) ways of arranging long and short syllables in a cadence of length six in Sanskrit poetic tradition, by Romaine on Wikipedia.

Roots and Wings

Make models to ground your math. Make connections to advance in higher math.

Make visual art and poetry with the beloved Fibonacci Numbers to connect with sequences. Arrange shapes as you explore visual proofs of theorems about sequences. Build hands-on and run-around models of recursion for a taste of "unplugged" computer science.

Words With Math Friends

Tell friends and family all about your math creations. Use these terms +"math" to find images, videos, and articles on the web.

- proof, visual proof, proof without words
- loop, recursive rule, explicit formula
- paradox
- infinite sequence, Fibonacci sequence
- algorithm, Smart Little People method
- Computer Science Unplugged

Interesting Choices

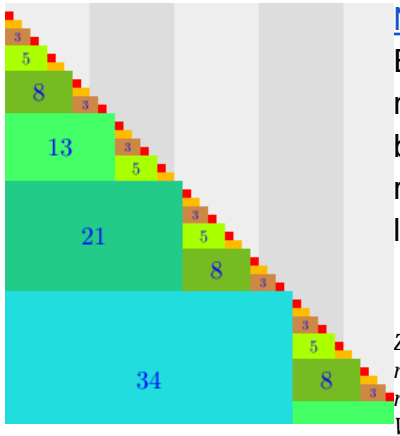
Mathematicians do many different things. What kind of math person will you be today?

Use counting and algebra or measuring and geometry to make the Fibonacci sequence. Choose abstract math to imagine the impossible at the end of infinity, or choose applied STEM to study realistic large numbers. Explore sequences from nature and culture, or design your own.

Toolbox

Physical (gray=optional)
Paper, graph paper, colored pencils, scissors, counters such as chips or beans, square blocks.

Virtual
Virtual math pad https://mathigon.org/polypad
A poetry tool https://visuwords.com/
~365k known sequences https://oeis.org/
Nature by Numbers https://youtu.be/kkGeOWYOFoA



NaturalMath.com/circles/
Extra activities, videos, math connections, books, and other resources for math circle leaders.

Zeckendorf theorem: natural numbers as sums of Fibonacci numbers, by Nomen4Omen on Wikipedia

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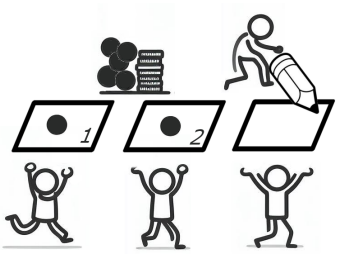
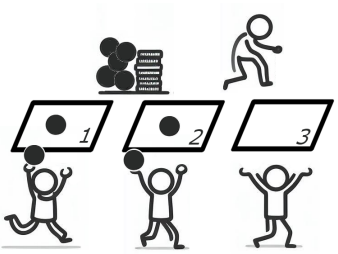
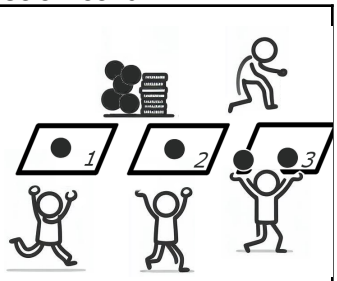
Math Circle: Fibonacci Day 11/23

0. Warm-up: The Fibonacci Crew

Have you ever seen videos of pros changing race-car wheels or making meals for big events? A crew gets a lot done quickly and precisely. Math people also imagine each operation as a person with a job. It's fun to imagine tiny characters running around your page and adding or multiplying numbers for you. That method of Smart Little People helps us think through the details. It's easier in practice than it is in words!

Assemble a crew of 4 math friends, or imagine tiny people. Prepare a box with index cards and counting chips. Write 1 and 2 on the first two cards (3, 4, 5, 6… will follow) and put 1 chip on each card.

		
INDEX Set down the next card. Write the next index on it. **COPY 1, COPY 2, ADD** Move in that order to three cards so that ADD stands by the latest card INDEX set.	**COPY 1** Take as many chips from the box as your card has. **COPY 2** Take as many chips from the box as your card has.	**ADD** Take chips from COPY 1 and COPY 2 with your two hands. Put the chips down on your card.

Repeat the whole algorithm again and again until you get tired or run out of chips.

Image: Gustavo Campas

1. The Big Letter and the Tiny Index

Fibonacci is a nickname of a 12th-century trader and mathematician. He loved the Hindu-Arabic number system he learned in North Africa from the Silk Road merchants. He published a big math book about that system in Europe. One of the most popular chapters in that book was about the sequence you just built. We have a tradition of writing the numbers from sequences a certain way. We use F for Fibonacci and a small index next to it, like we did with index cards. Write down the next few Fibonacci numbers.

$F_1=1$	$F_2=1$	$F_3=2$	$F_4=3$	$F_5=$____	$F_6=$____		

When you make up your own sequences, use your initial or your favorite letter to index them.

2. The Fib Poems

Long before Fibonacci popularized this sequence in Europe, the creators in India used these special numbers to make songs and poems. Here is a modern Fib Poem called *Word Crunching* by Brian Bilston. Do you see how this poem works? Try to write your own Fib poem to celebrate Fibonacci Day!

I
wrote
a poem
on a page
but then each line grew
by the word sum of the previous two
until I started to worry at all these words coming with such frequency
because, as you can see, it can be easy to run out of space when the poem gets all Fibonacci sequency.

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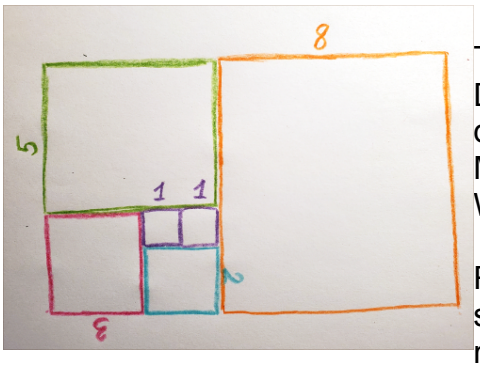
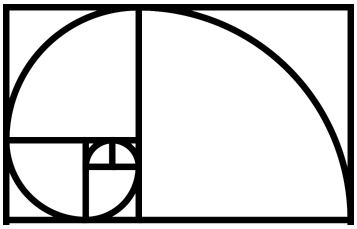
3. The Nautilus Spiral and the Stacked Squares

There are many ways to make the Fibonacci sequence. We could count, or draw and measure. To start, draw a small square in the middle of a big sheet of paper (plain or graph). Draw another square of the same size next to it. Write 1s at the tops of your squares. These are your units for measuring. And then:



TURN your paper a quarter-turn (90 degrees) counter-clockwise.
DRAW another square to the right of your design, as tall as your whole design is.
MEASURE the side of that square in your units.
WRITE the measure on top of your square.

Repeat the whole algorithm again and again till you get tired or run out of space on your paper. Pause to admire the stacked square design you made. What do you notice about the design?



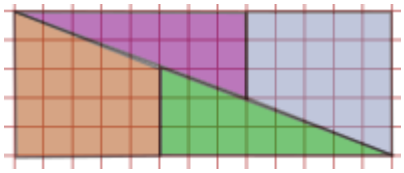
The grid you just made appears in nature and art a lot. It is beautiful to many people. Here is one example of a design you can make with this grid. Hint: it helps to turn your paper after you draw each quarter-circle.

Imagine filling your design with unit squares. How many will you need? There are two ways to look at this: as the sum of small squares or as one big, huge rectangle. Whenever there are two ways to look at identical numbers, math people call that an identity.

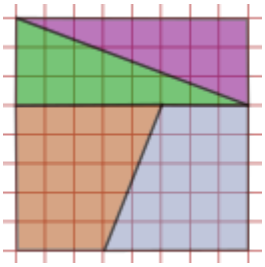
1*1 + 1*1 + 2*2 + 3*3 + 5*5 + 8*8 = 8*13

$$F_1^2 + F_2^2 + \ldots + F_n^2 = F_n F_{n+1}$$

4. Fibonacci Stole My Chocolate!



Here is a party trick to amuse your friends on Fibonacci Day, called Chessboard Paradox. This works well with large-cell (half-inch or so) graph paper. To prepare, cut out an 8x8 square. Split it into two triangles and two trapezoids, as shown on the right, and also cut them out. Arrange these shapes on your graph paper to make a rectangular chocolate bar. Ask people to count how many small squares of chocolate are in your bar. Next, rearrange the same shapes on your graph paper to make a square chessboard. Ask how many unit squares are in your chessboard. The answer will be smaller! Where did the chocolate go?



$$5 \times 13 \Rightarrow 8 \times 8$$

There are three consecutive Fibonacci numbers in play here. The same trick will work with any four consecutive Fibonacci numbers. Try that for yourself sometime!

5. Make and Find Your Own Sequences

- Make up your own rules for a crew of friends and build your own recursive sequence.
- Grow your math eyes too see sequences in nature and art, architecture, poetry, and everywhere you go.
- Sometimes things grow or are built by mathematical rules. Other times we imagine interesting patterns, as if seeing shapes in the clouds. Psychologists call that pareidolia. Try to tell which it is!

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~ Examples ~ Questions ~ Problems ~ Stories ~ Conjectures ~ Models ~ Art ~ Formulas ~ Graphs ~

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