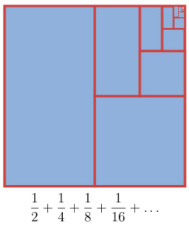
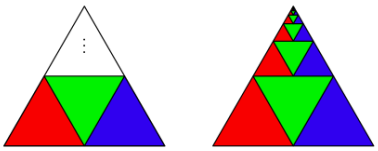
Infinite Series

1 Picture Proofs of Geometric Series

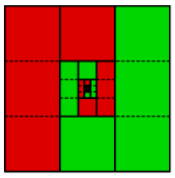
1. How does this picture help you find the sum of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ (where the $\cdots$ means you keep adding up numbers in this pattern forever?



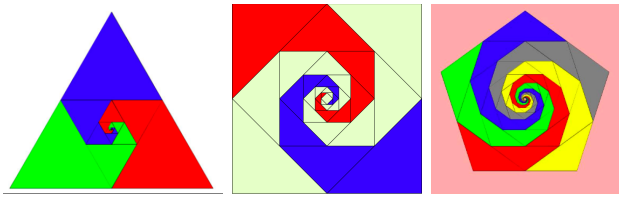
2. What infinite sum can you add up with this picture, and what does it sum to?



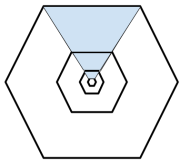
3. What infinite sum can you add up with this picture, and what does it sum to?



4. Use each figure to write down an infinite sum and add it up. Can you generalize to other figures?



5. Suppose each small hexagon has area that is $\frac{1}{7}$ of the area of the next larger hexagon. What infinite sum can you add up with this picture and what does it sum to?



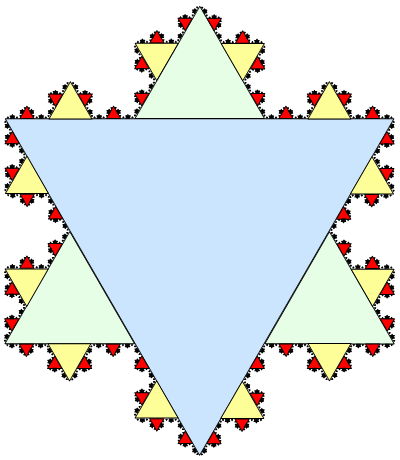
6. What other sums can you find with similar pictures?

2 Sums of Series

7. What is $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots$.
8. Find $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\cdots$
9. Find $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\cdots$
10. Find $\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\frac{16}{625}+\cdots$
11. Compute the value of $\frac{1}{1}+\frac{2}{2}+\frac{3}{4}+\frac{4}{8}+\frac{5}{16}+\frac{6}{32}+\cdots$. Can you use the answer to a previous problem to help?
12. Compute $2+\frac{2}{10}+\frac{2}{100}+\frac{2}{1000}+\frac{2}{10000}+\cdots$. Can you write out this series in decimal form?
13. Write the following decimals as fractions
    (a) $0.555555\cdots$
    (b) $1.3333333\cdots$
    (c) $0.24242424\cdots$
    (d) $0.4285714285742857142857\cdots$
14. Did you know that $0.99999\cdots$ is the same number as 1? Prove it!
15. Find the sum of the series $\frac{1}{10}+\frac{2}{10^2}+\frac{3}{10^3}+\frac{4}{10^4}\cdots$
16. What is the sum of $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}\cdots$? Hint: turn products of fractions into differences of fractions.
17. What is the sum of $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\frac{1}{4\cdot 6}\cdots$?
18. Find the sum $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{4}{5!}+\cdots$

3 Koch Snowflake

The Koch snowflake is made up of an infinite number of equilateral triangles. To build it, start with a triangle with side length 1 and keep adding triangles that have $\frac{1}{3}$ the side length of the previous triangles. Suppose that the side length of the original blue triangle is 1 unit.



19. What are is the perimeter of the figure after the green triangles are added?
20. What is the perimeter after the yellow triangles are added?
21. What is the perimeter one the red triangles are added?
22. In the limit, what is the perimeter of the entire snowflake? Is it finite or infinite?
23. What is the area of the blue triangle? Hint: the area of an equliateral triangle with side length a is $\frac{\sqrt{3}}{4}a^2$.
24. What is the area of each green triangle? Each yellow triangle? Each red triangle?
25. How many green triangles are there? Yellow? Red?
26. Write the total area of the Koch snowflake? Is it finite or infinite?

4 Coin Flipping

27. Two players alternately toss a penny, and the one that first tosses heads wins. What is the probability that:

    (a) the game never ends?

    (b) the first player wins?

    (c) the second player wins?

28. On average, how long do you have to wait when flipping a coin before you get the first head? Hint: Let x stand for the average number of flips you'll have to wait.

29. What is the probability of getting HH in two flips? HT?

30. How many times do you have to flip a coin on average to get HH?

31. How many times do you have to flip a coin on average to get HT?

32. If you bet on HT happening first, and I bet on HH happening first, what are your odds of winning?

33. What is the probability of getting HHH in three flips? HTH? HHT?

34. How many times do you have to flip a coin on average to get HHH?

35. How many times do you have to flip a coin on average to get HTH?

36. How many times do you have to flip a coin on average to get HHT?

37. If you bet on HTH happening first and I bet on HHH, what are your odds of winning?

38. What about HHH vs HHT?

39. And HTH vs HHT?