Chapel Hill Math Circle
Session 6 – December 2, 2023: Fun with Angles
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com
Supplies needed: rulers, protractors, blank and graph paper, colored pencils

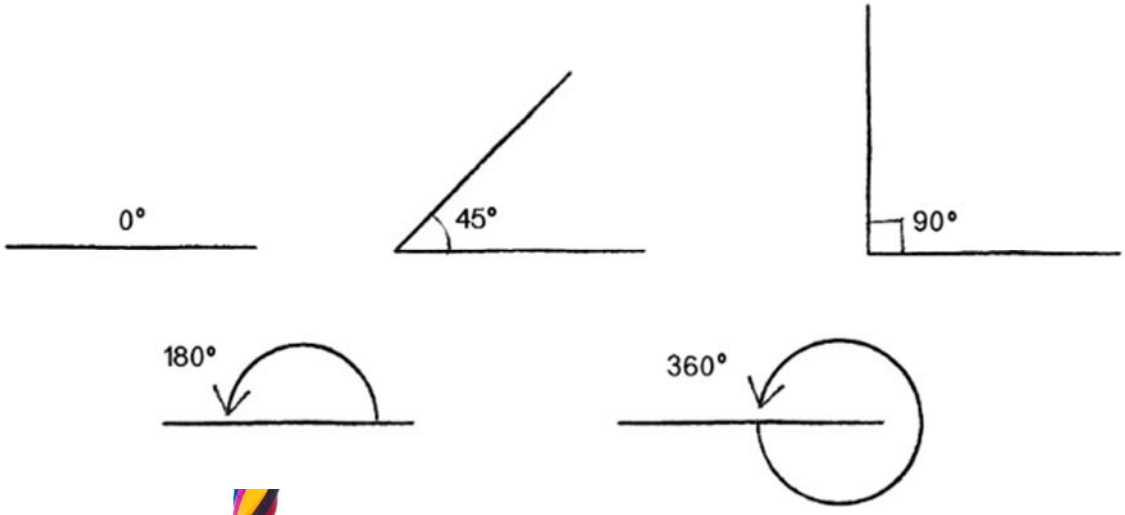
Chapel Hill Math Circle

Happy December and welcome to Chapel Hill Math Circle! We're glad that you are here to have some fun! Let's pursue angles!

Angles

What is an angle? Last time those who were at math circle came up with great descriptions. We will define an angle as the **space formed inside two intersecting lines** or line segments. We typically measure the space by a number called the **degree** that goes from **0 to 360**. Khan Academy[1] has a nice summary image.

I like to compare angles to the opening of a book. You might have heard of obtuse angles, acute angles, right isosceles triangles, equilateral triangles, and more. Don't worry about these names for now. And it's okay if you haven't heard of them!



A right angle or 90 degree (90°) angle is like the corners of a piece of paper. Degrees numbered between 0 and 90 measure how open a book is.

[1] khanacademy.org/computing/computer-programming/programming-natural-simulations/programming-angular-movement/a/angles-and-units, accessed Nov. 8, 2023

Questions

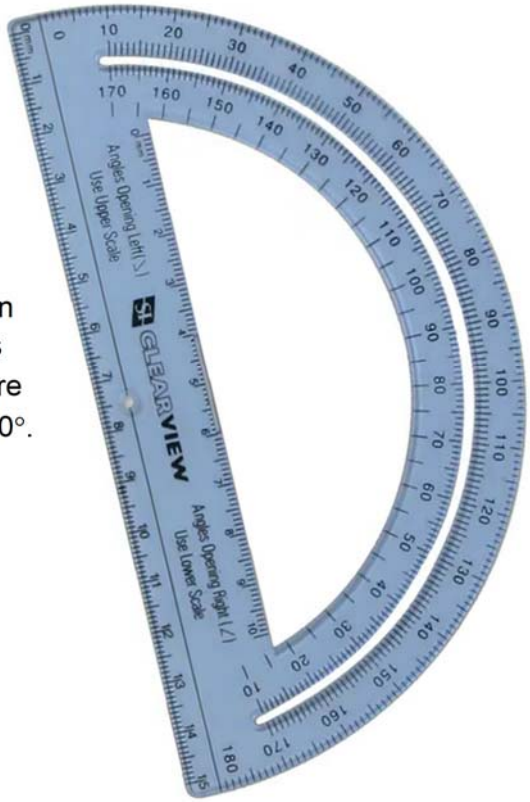
Why do we use numbers like 90, 180, and 360? What is an angle like 360°+360° = (300+300+60+60 = 600 + 100+20 = 720) 720°? Or 360° added together many times? Please discuss with a friend and jot some notes down. Leave space to add more notes when we discuss this in a few minutes.

Notes on angle questions

Protractors

Last time we learned to use a protractor[2]? Here's what learned with the help of a picture from cuemath[3].

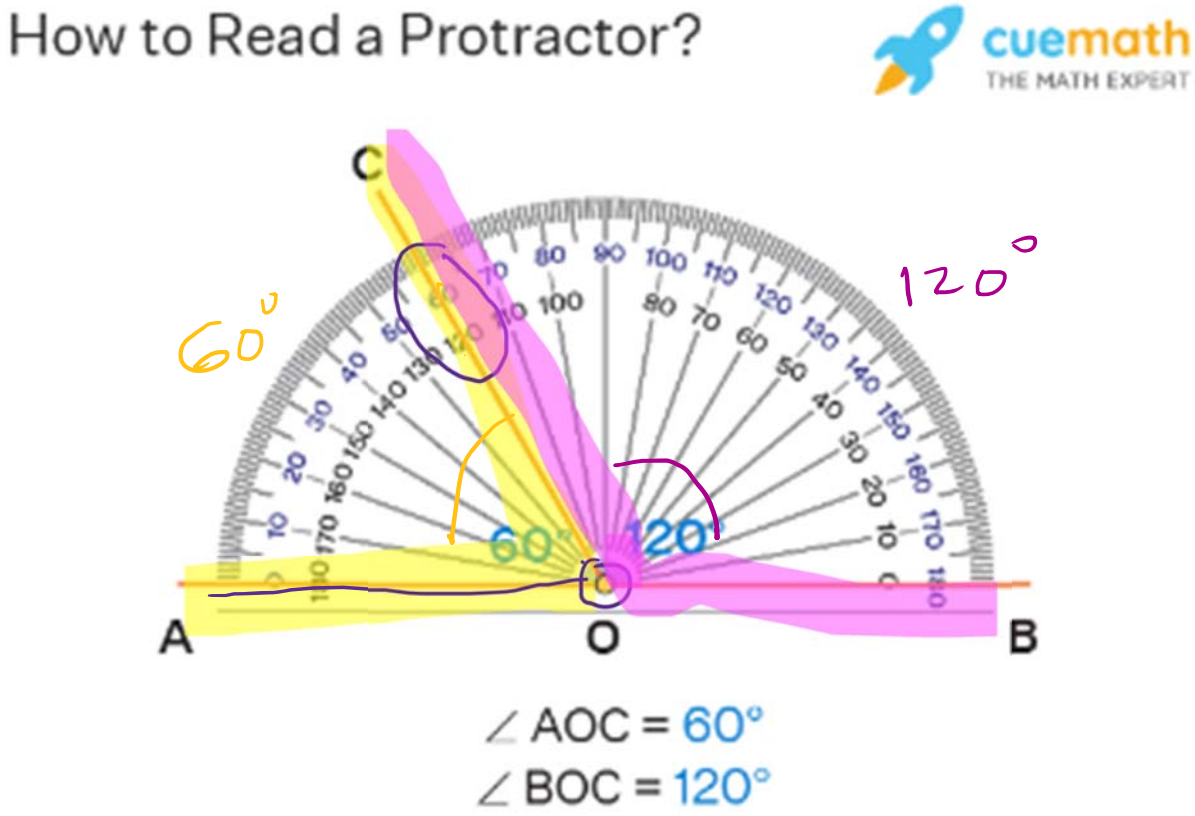
- Use a ruler to make an angle.
- Line up the center hole (marked O below) with the angle's vertex and have one side of the triangle go along the line at the bottom of the protractor (A to O or O to B). Look at what numbers the other line goes through. In the example below, line OC goes through both 60 and 120.
- Angles smaller than a right angle will be between 0 and 90 degrees. Therefore AOC, which is less than a right angle will be 60°. BOC, which is more than a right angle, will be the bigger number, 120°. Note that a straight angle is 180°.



[2] The picture of the protractor is from Fisher Scientific, fishersci.com/shop/products/clearview-transparent-plastic-protractor/S90491A, accessed Nov. 8, 2023

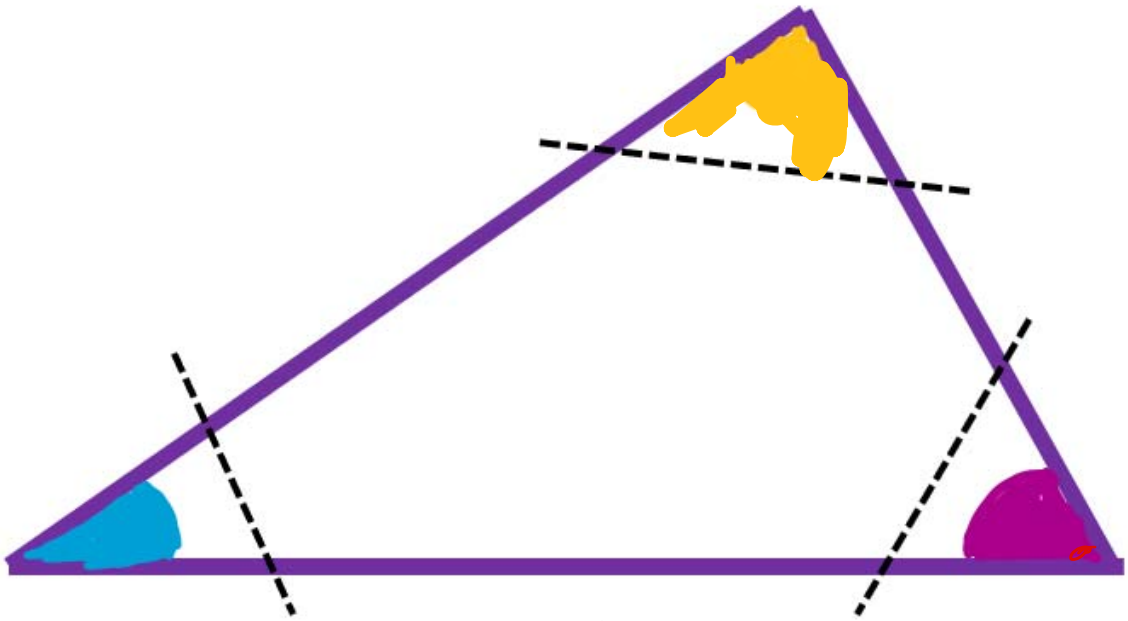
[3] from cuemath.com/geometry/protractor, accessed Nov. 15, 2023

A straight angle is $180°$ … and the angles of a triangle sum to $180°$



We also learned that the angles of a triangle area always a straight angle; since we call a straight angle 180°, we can say that the angles of a triangle add up in measure to 180°. Those who were here last time did the experiment below.

Draw a triangle and color the three corners (vertices). Then use scissors to cut out the three angles. You can even just rip; the cuts don't have to be clean. You should have something like the three pieces at the bottom. Shove them together and you will find that no matter what triangle you have started with, you have the three pieces forming a line. This shows that the angles of a triangle add up to a $180°$!



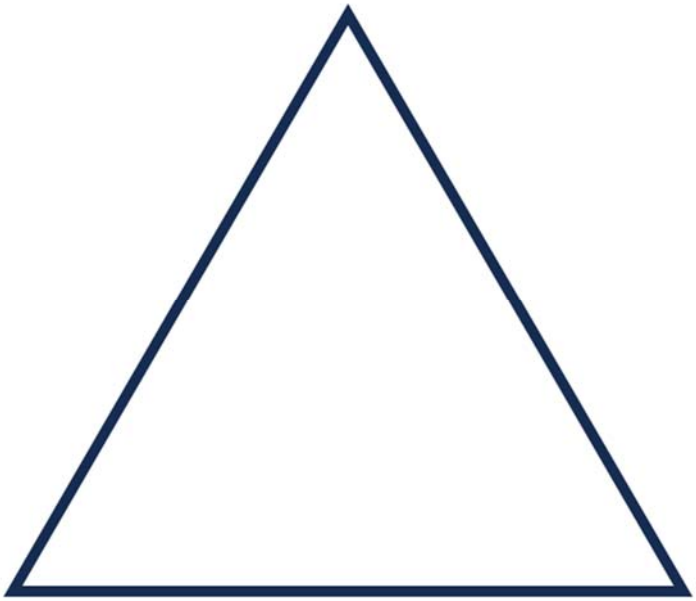
Let's measure polygons

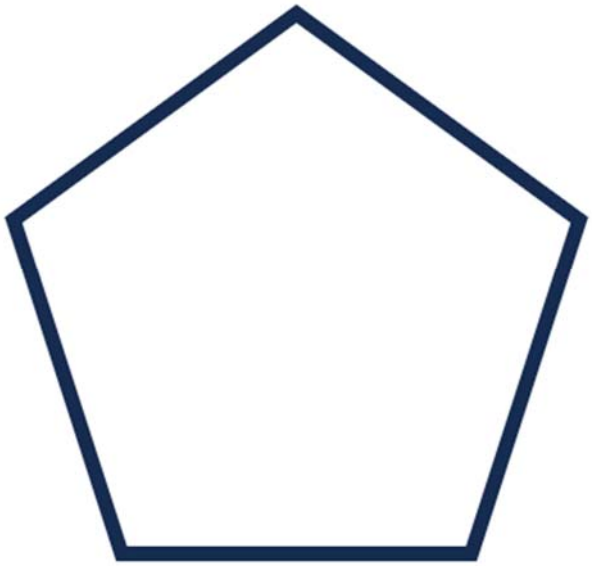
Today's main task is to measure polygons. On the next page are the first few regular polygons, a 3-gon (triangle), 4-gon (square), 5-gon (pentagon), 6-gon (hexagon), 7-gon (heptagon), and 8-gon (octagon). They are **regular**, which means that all the sides and all the angles are the same.

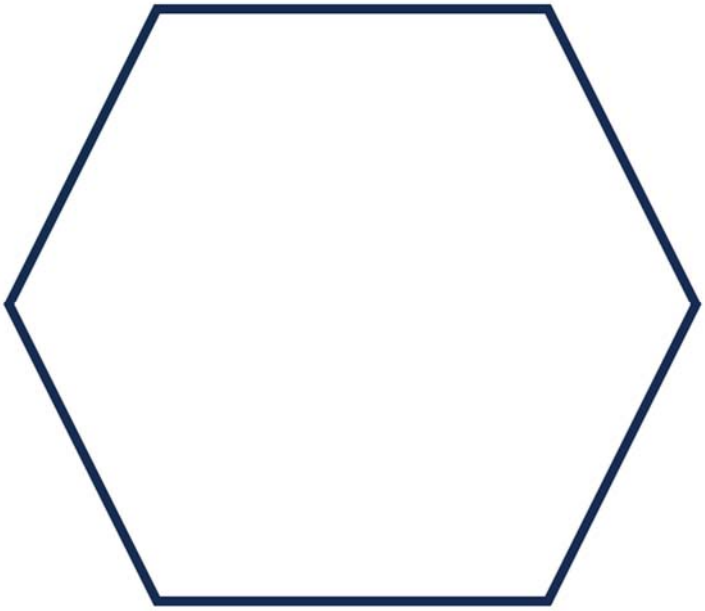
Here's what we will do. We will fill out the table on the right. Can you figure out what the sum of the angles of any n-gon is based on the number of angles (n) without measuring?

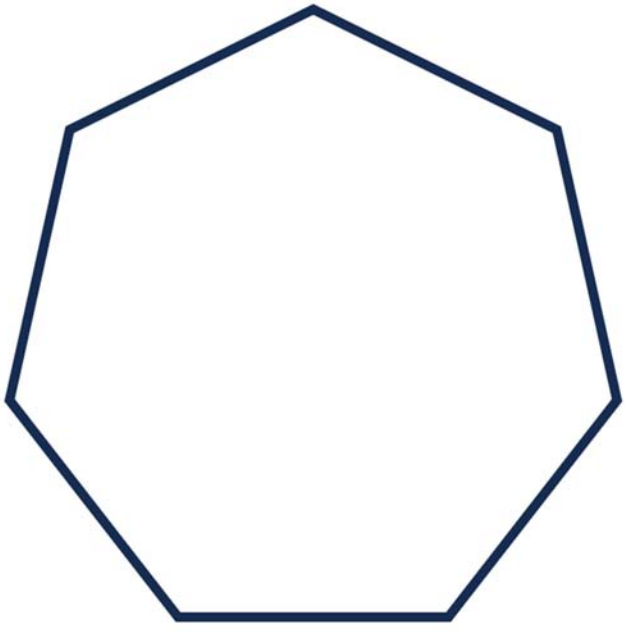
I filled out the first entry; we know that the angles of a triangle add up to 180°. Since all the angle are the same (since it's a regular triangle, also known as an equilateral triangle), we have to find a number that added to itself 3 times is 180. I'll let you figure it out. One way is to guess and check. 10 sounds too low (yup – 10+10+10 is just 30) and 100 sounds too high (3 100s is 300). Guess something in between and if you are wrong, "fix" your guess till you are right.

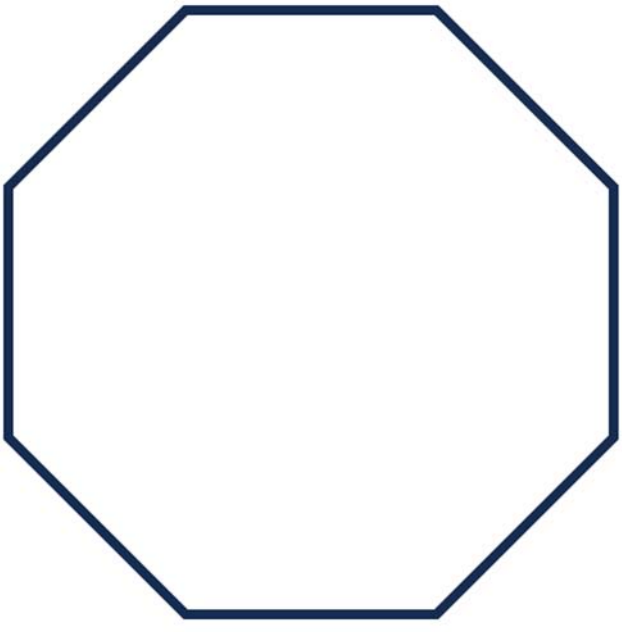
Angles of a Regular n-gon		
n	Sum of angles	Each angle
3	180°	
4		
5		
6		
7		
8		





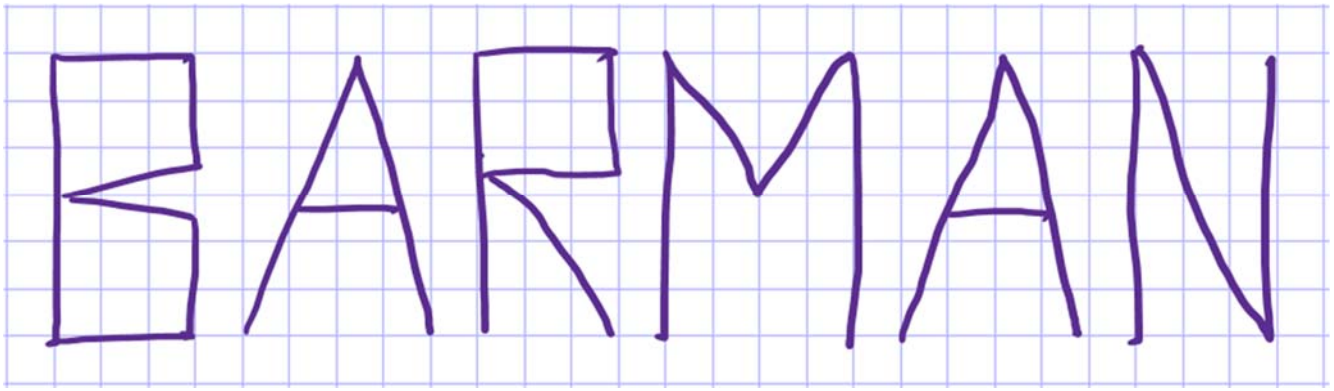




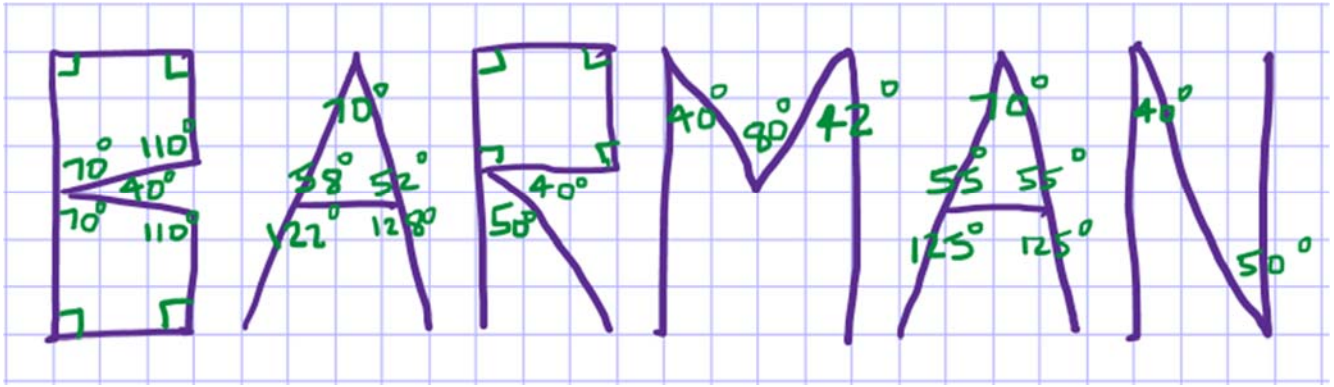


Play with your protractor

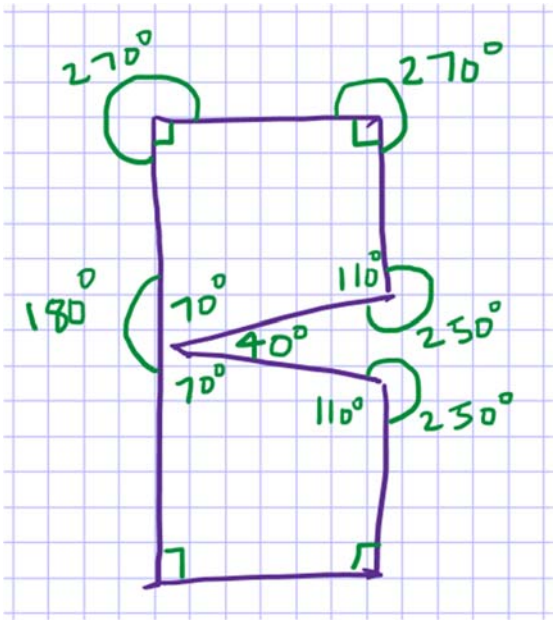
Here is a fun activity with graph paper which has a grid of light boxes. Carefully and neatly write your first name or your last name on it but try to make your letters as straight, not curved, as possible. Here I have written my last name:



Then use a protractor and measure as many angles as you see, like I did:

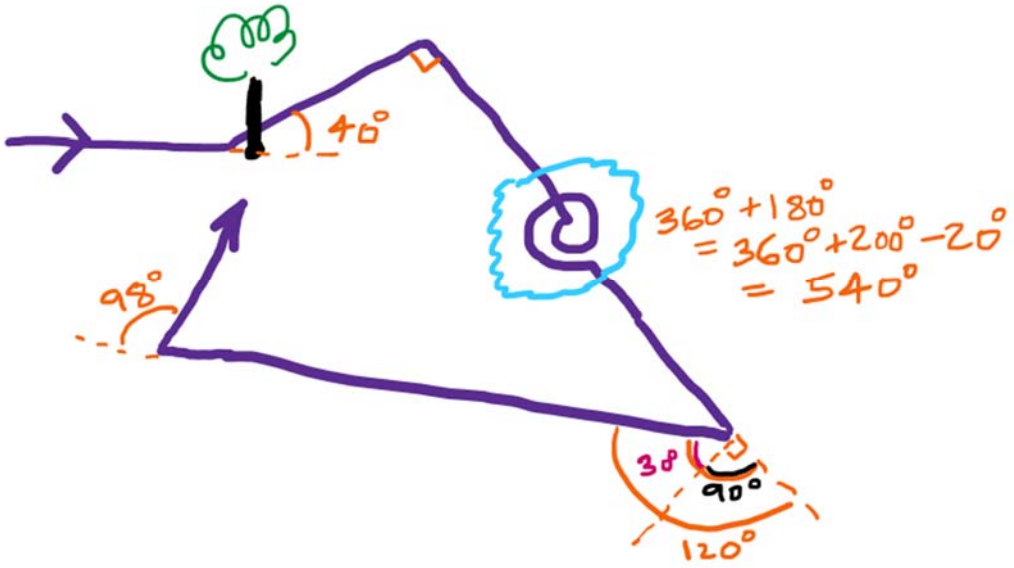


You can even identify what are called reflex angles or angles that are kind of outside of the angle that you might first have thought. I've included a closeup of the "B" in my name where I did that.



Tell a story

We probably won't have time today but here's a fun activity to try at home. Make a diagram and then tell a story about that diagram using angles. Here is an example.

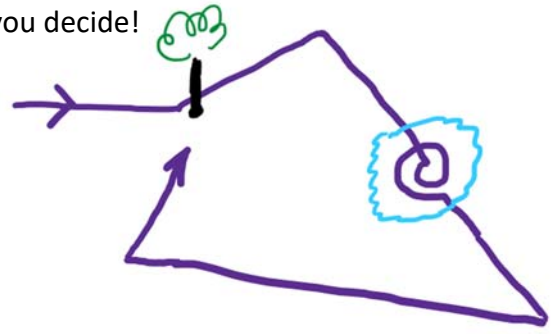


I was walking with my aunt to an art museum one day. We were walking through a lovely park when our path came straight to a beautiful and large tree. We followed the path which turned 40° to avoid the tree. I was hungry and after a few minutes I smelled pizza. We decided to investigate and made a right angle toward the smell.

Unfortunately the path had a wet spot and I slipped (my aunt was okay because she had boots on), turning all the way around one and a half times or 540°. But all's well – I didn't get hurt and it turned out to be fun!

But after a short while we realized that we had to get home because we forgot that a friend would be visiting. We turned 120° and headed toward home. Along the way our friend called and said they would be late. My aunt and I wanted a break and could see that large tree that we had passed again this time to our right. We turned 98° toward it so that we could rest in its shade before heading home.

You can draw the diagram with our without writing in the angle measurements, but do use the angle measurements in your story. Here to the right is the diagram without writing in the angles. I think if you can add them to the diagram, especially if you use a different color, it makes it easier to follow the story, but you decide!



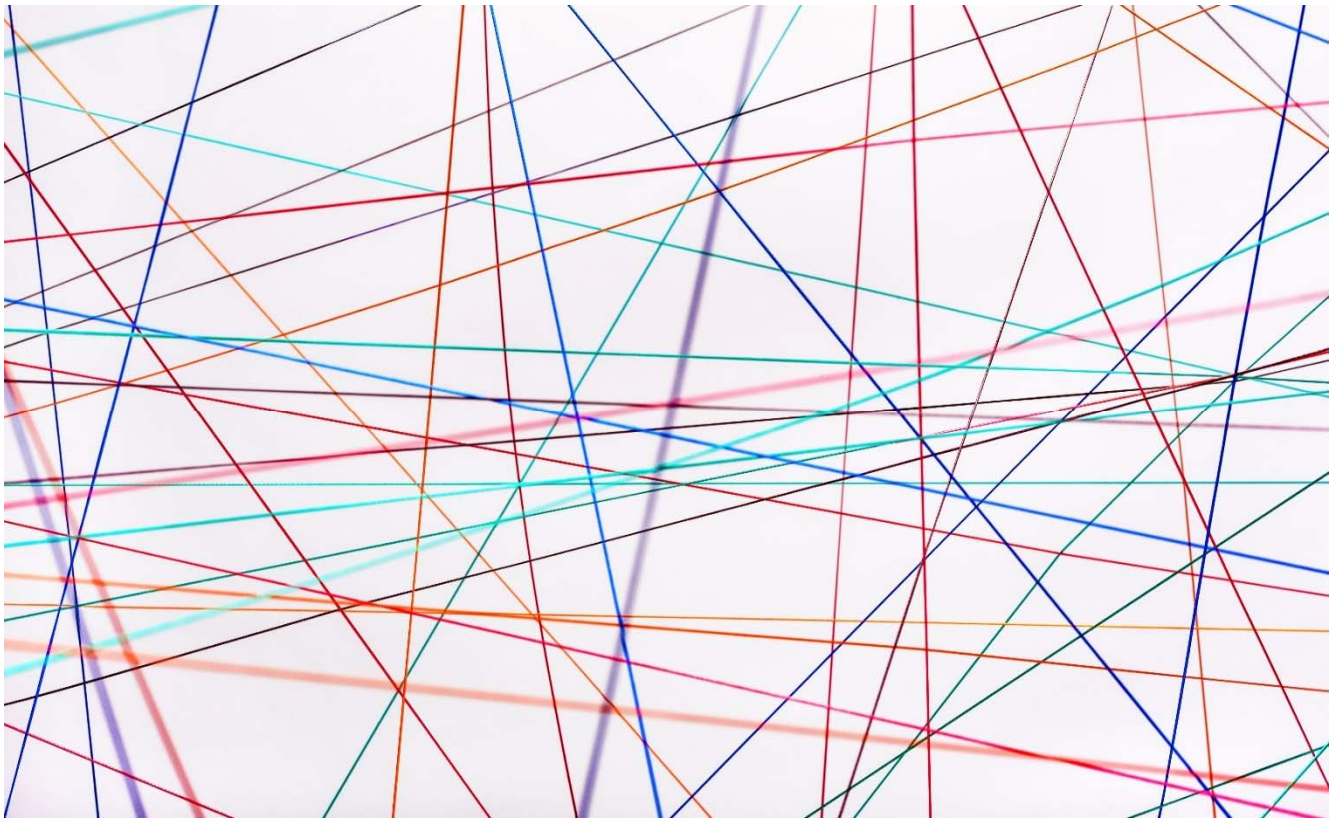
Here are a few questions to think about.

- The triangle is considered a strong shape. See if you can figure out why. Do you see triangles in roofs, bridges, playgrounds, and other places? Make sketches of angles and triangles.
- If you visit an art museum, sketch a painting or sculpture that has interesting angles / triangles. Or look at some art that your family has on your walls or in a book.
- Look at a bicycle's frame.
- Find angles in toys.
- Make up your own angle measure. Instead of 180 degrees being straight, how about 100 somethings or 10 somethings?

I hope you had fun! See you next time!

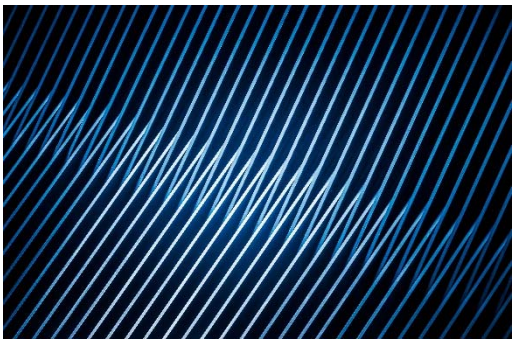
Have Fun!
Mr. Barman

P.S. Let your parents know that registration for Math Kangaroo, a fun math competition in March, is open. I encourage them to check out **mathkangaroo.org**. We're offering it at my school, **trianglemathinstitute.com**, and if we have space after our own students register, we may be able to open some seats in a week or so. UNC and other schools are also offering it.



Notes for Parents

If you were with us last time, I have more detailed information on calculating diagonals that I developed with my Stanford math circle students. Feel free to contact me if you would like me to email you that information.



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Angles

Angles and triangles are pervasive. If you have a chance soon or, if not then over the holidays, I'd love for you to take your child to an art museum and explore how artists integrate geometry into their creations. One of the homework problems is just that – to look around and see where your child can find angles in particular. We are lucky to have such great museums such as the Ackland walking distance from math circle, the NC Museum of Art in Raleigh, and Duke's Nasher.

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Angles in an n-gon

I wanted to have students bridge their technical finesse with their protractors and their intuition to address the problem of angles in an n-gon. The angles of a triangle add up to 180°. If we want to know how many degrees any quadrilateral (4-gon) has, we can readily reason that it consists of 2 triangles and so the angles must add up to 180°x2 = 360°. In general, and I trust your child may find this, an n-gon can include n-2 triangles.

To the right is a summary of the sum of the angles and the average angle of several n-gons. We haven't discussed the idea of an average so for now I'm just having the children work on regular polygons.

n	# triangles	Total degrees	Avg degree per vertex
3	1	180	180/3 = 60
4	2	180x2 = 360	180x2/4 = 90
5	3	180x3 = 540	180x3/5 = 108
6	4	180x4 = 720	180x4/6 = 120
7	5	720+180 = 900	180x5/7 = 128 4/7
8	6	900+180 = 1080	180x6/8 = 135
N>2	N-2	180x(N-2)	(180x(N-2))/N

I don't use multiplication or division with 1st or 2nd graders as I want them to really exercise addition and subtraction. Some of the students may have been exposed to division (yikes!) and can solve the problem of finding the angle of an n-gon by dividing the total degrees by the number of vertices. But I hope that most if not all use a technique of estimating and fixing that is good to know in many contexts.

I gave an example in the notes but wanted to share another with you. When your child works out a 7-gon, they will find that the total number of degrees is 5 180s. I bet they'll also realize it's just 180 degrees more than what they got for the 6-gon; if not, then please encourage them to this computational shortcut.

Do I expect them to get 128 4/7 degrees for the angle of one vertex in a regular 7-gon? No. I expect them to use their protractors and find that the answer is around 125-130 and then find that 7 128s comes close and so does 7 129s. If that level of precision is too much then answering that the measure is around 130 is "good enough".

This skill of problem solving "on the back of an envelope" is an important one. Sometimes smart children focus on "exact" algorithms but then they don't always have a sense of what the answer really means. This is why, even if your child knows division, I would love for you to get them to "solve" this problem by estimating and fixing till the answer is "close".

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I hope that the exercises I have suggested are fun and solidify your child's angle measuring and observation skills. If you don't have a protractor, they are readily available at stores like Michael's and online. I hate recommending plastic but plastic is best for the transparency and in any case is by far the most common; a good protractor can last a lifetime. Thanks for coming to math circle - best wishes!

Dilip

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