Two is the Best Number (Part 3)

1 Warm-Up

1. In this problem there is a long straight road.

   (a) A cricket jumps along the road. Every jump takes him 1 meter to the right or to the left. Could it be that in 21 jumps he will end up exactly where he started? Explain.

   (b) A brown cricket and a green cricket are sitting on the road 9 meters apart. They start jumping at exactly the same moment. On every jump, each of them leaps 1 meter to the right or to the left. Whenever one jumps, the other jumps too, at the same time. Can they ever land on the same spot simultaneously? Explain.

   (c) A brown cricket and a green cricket are sitting on the road 15 meters apart. They start jumping at exactly the same moment. On every jump, each of them leaps 3 meters to the right or to the left. Whenever one jumps, the other jumps too, at the same time. Can they ever land on the same spot simultaneously? Explain.

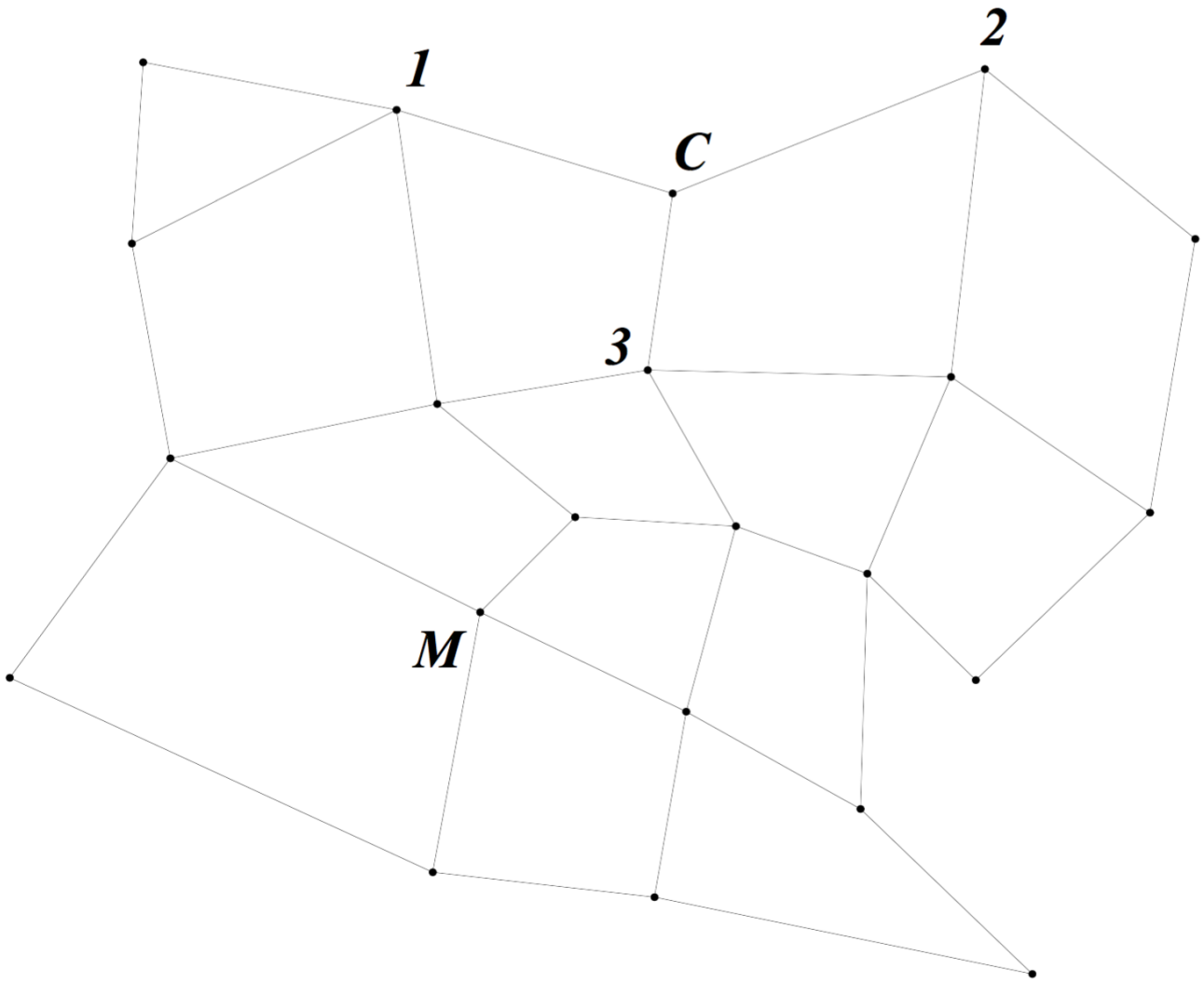
2 Alien Captors

2. Aliens have imprisoned 10 math circle participants. The aliens say, "You will be placed in a single file line, facing forward and I will put a hat on each of your heads. The hat will be colored either white or black. You will be able to see the hats of everyone in front of you, but you will not be able to see your hat or the hats of the people behind you. I will begin by asking the person at the back of the line to guess their hat color. If the guess is correct, that person will get a cookie. If the guess is wrong, that person will be eaten. Then I will ask the next person in line, and so on. You are only allowed to say the single word 'black' or 'white' when it is your turn to speak, and otherwise you are not allowed to communicate with each other while you are standing in line. Although you will not be able to see the people behind you, you will know (by hearing) if they have answered correctly or not."

   The prisoners are allowed to chat for a few minutes before their ordeal begins. What is the largest number of prisoners that can be guaranteed to survive?

3 Cat and Mouse

3. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.



Initially, the cat is at the point labeled C; the mouse is at M. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?

The cricket problems is from *Mathematical Circle Diaries, Year 1* by Anna Burago. This version of the famous prisoner hat problem is taken from Alex Gendler's Ted Talk on Can you Solve the Prisoner Hat Riddle. The cat and mouse game is from James Tanton's book *Solve This!*