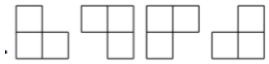
Tiling with Dominos and Triominos

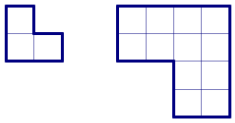
Tiling with bent triominos

A *size-1 bent triomino* is a 2×2 board from which one corner has been removed.



1. Is it possible to tile a $2^n \times 2^n$ board from which one square has been removed with size-1 bent triominos? Does it matter which square has been removed?

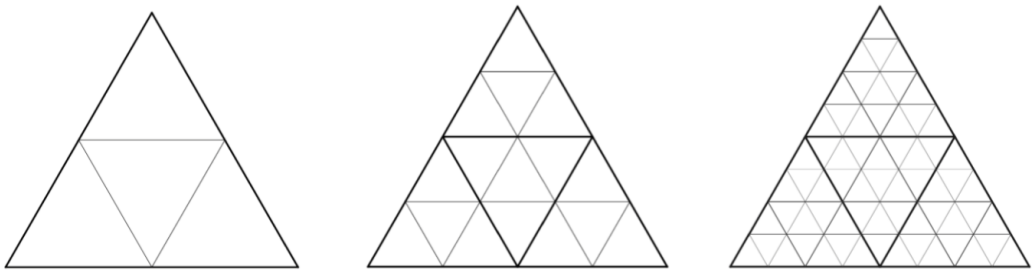
   A *size-n triomino* is the shape you get when you remove one quadrant from a $2n \times 2n$ square. In the figure below, a size-2 triomino is on the right.



2. Can a size-5 bent triomino be tiled by size-1 bent triominos? Can a size-2024 bent triomino be tiled by size-1 bent triominos?
3. Which square boards of size $n \times n$ from which one square has been removed can be tiled with size-1 bent triominos? (Hint: The 5×5 board is special.)
4. Which $n \times m$ rectangular boards can be tiled with size-1 bent triominos?

Tiling triangular boards with triangular triominos

Write $T(n)$ for a triangular board of side-length 2^n which is subdivided into equilateral triangles each of side length 1. If a triangle shares one (or two) of its sides with the large triangle, then it is called an *edge triangle*. If it shares two of its sides with the large triangle, then it is called a *corner triangle*. A (triangular) triomino is a *tile* consisting of three adjacent triangles.



5. For which n is is possible to tile the remaining board with triangular triominos after any (one) corner triangle is removed from $T(n)$?

6. For which n is is possible to tile the remaining board with triangular triominos after any (one) edge triangle is removed from $T(n)$?
7. For which n is is possible to tile the remaining board with triangular triominos after all the corner triangles and any other (one more) triangle is removed from $T(n)$?
8. For which n is is possible to tile the remaining board with triangular triominos after any (one) triangle not adjacent to a corner triangle is removed from $T(n)$?

How many ways?

9. In how many different ways can a rectangular $2 \times n$ board be tiled with dominos? Start with small numbers n, then generalize.

Show your true colors

10. Is it possible to tile a 5×5 square board with dominos?
11. Is it possible to tile with dominos a 5×5 board from which one square has been removed? Does it matter which has been removed?
12. Is it possible to tile with dominos an 8×8 square board from which two opposite corners have been removed?
13. Find all squares on an 8×8 board such that when one of these squares is removed, then the remaining part can be tiled with (straight) triominos.

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