Chapel Hill Math Circle
Session 4 – October 26, 2024:
Voting
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com

Supplies needed: paper, pencil

Welcome to Math Circle session 4! It is almost November and we are in the midst of election season. We have people running for the U.S. President, Congress, and area offices like mayors and city council. If you get a chance, tag along when your parents vote and maybe they'll even let you help them vote, perhaps filling in some ovals for choices they would like to make.

It turns out that math is useful in voting! There was a man named Kenneth Arrow[1] who came to the conclusion that if we have three or more choices, voting can't ever be as fair as we wish it could be. We will briefly look at what he said and then try our hand at various kinds of voting.

Why do we vote?

Sometimes not many people want a job and anybody qualified can perhaps take the job. But if many people want one job we sometimes vote. But we also vote for ideas. Maybe your school decides that they want students to help choose what fruits will be in the cafeteria or what field trip your class will go on.

When there are more than 2 choices, voting can get complicated

Maybe a baseball team is going out for a party and the captain wants students to vote whether they want to go out to a pizzeria, noodle shop, or food court. Maybe you get to go on a class trip and have five choices. Maybe your parents want to go out to see a movie and there are three choices. We will explore why it can get complicated when there are more than 2 choices. Can you think of some examples from your life where people had to pick from at least three choices?

What makes voting fair?

There was an economist named Kenneth Arrow. In the 1950s he came up with a list of 4 things that make voting fair.

[1] The picture of Dr. Arrow, who won the Nobel prize in Economics in 1972, is from the Nobel Foundation archive and retrieved from nobelprize.org/prizes/economic-sciences/1972/arrow/facts/, accessed Oct. 25, 2024.

1. **No dictator** A dictator is a king or somebody who does what she wants without anybody stopping them. Dr. Arrow argued that in a democracy we must make sure that our elections show the choice of more than one voter; one person should not decide an election.
2. **Unanimity** That's a fancy word for saying that if John runs against Mary and all voters vote for Mary, Mary should win.
3. **Universality** There should always be one winner.
4. **No spoiler** If Saraswati runs against Vadem and she beats him, it should not matter how voters voted for a third person, say Naomi.

There have been other ideas too like if most people rank Joakim as their top choice, that candidate should win. But the surprising result is that no matter what rules we use, **Arrow's Theorem** says that no voting system with at least 3 candidates can be "fair" and satisfy all of the 4 conditions. If you want to explore this a bit, your parents have some references that they can use to learn more. But for now let's experiment with voting.

Let's vote between two things – field trips

Let's think of two places you might go on a school trip – maybe to eat and tour the museum at Videri Chocolate Factory in Raleigh; enjoy outdoor and indoor art at NC Museum of Art in Raleigh; tour Raleigh-Durham Airport in Morrisville; learn the history of the American Tobacco Campus in Durham; find out how UNC in Chapel Hill was the first state university established; see how the farmers' market is organized in Carrboro; or enjoy some other place. Let's as a class pick two things to vote on.

Now let's vote. Let's learn some words. If we have an even number of people and everybody votes, one choice will win unless there's a tie, right? We vote according to **simple majority**. Can you think of any time when everybody might not go along with a simple majority choice? What if Sally is allergic to chocolate? What should the class that votes for Videri do? Should they have even considered Videri as an option?

Now let's vote on three things

Let's vote for one thing to do – go to the chocolate factory, art museum, airport, American Tobacco, UNC, or farmers' market. Let's vote!

What happened? Did we get a winner? If we did, how popular was the choice?

Let's find out what the simple majority would be. If there are 6 people and 4 or more vote for the choice, that's a simple majority since 4 is more than half of 6. Let's practice some halving to see what a simple majority might be for different numbers of people.

Take a few minutes and figure out how many people would have to vote for something to get a simple majority when we have different group sizes. I filled out a few entries in this table – please fill out the rest.

Simple Majority		
Total Number	**Majority**	**Why?**
0	0	If there's nobody voting, no votes makes a majority!
1	1	Whatever the voter says, goes. It's like a dictatorship except it's just one person.
2	2	If each choice gets 1 vote, no choice is more than half. Half of 2 is 1. Here a majority is all votes.
3	2	1+1 = 2 and 2+2 = 4, so half of 3 is more than 1 – at least 2 will be the majority.
4	3	If one choice gets 2 votes, that isn't a majority – half of all votes but not more than half go to it. Half of 4 is 2 because 2+2 = 4. So 3 (or 4) is the majority.
5	3	2 isn't a majority since 2+2 is less than 5. 3 is a majority since 3+3 is more than 5. So 3 (or 4 or 5) forms a majority.
6		
7		
8		
9		
10		

Did you find any patterns? Do the patterns have anything to do with parity – whether a number is even or odd?

When we vote, it's possible that everybody votes the same way. That would be a **unanimous** choice. Do you know *Winnie the Pooh*? If Eeyore runs against Pooh Bear for most cheerful character, everybody, even Eeyore, will vote for Winnie the Pooh; he certainly would win unanimously.

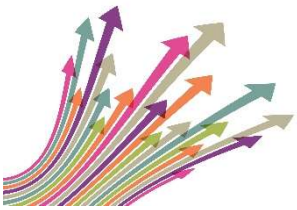
When we vote with two choices, assuming everybody votes and votes one way or the other, one of three things can happen:

- A choice is made unanimously. Hurrah! We're all happy!
- A choice is made with a majority. Almost half the people may not be so happy.
- There is a tie – as long as there are an odd number of voters.

What would you do if there were a tie? How would you work if you were voting in a small group, like your class or our math circle, to figure out what choice to make? How would this be different if you were working with a large city like New York?

But what are the possibilities if there are more than two choices?

Did you find that it's possible to win without a majority? If 10 people are voting and there are 6 choices A, B, C, D, E, and F, could 3 people vote for A, 1 for B, 2 for C, 1 for D, 2 for E, and 1 for F since 3+1+2+1+2+1 = 10? Maybe there are 3 airport enthusiasts and nobody else likes the idea of touring an airport, yet that idea wins. In this case, the winning choice is not one of majority but of **plurality**; the winner gets the most votes but that may not be the majority. (In the case where a choice wins by majority, that's also by plurality.)



Are there problems of winning with a plurality that is not a majority?

Let's continue with the example of the field trip. Let's add another option. Three Martians are visiting to make the class become a class of thirteen and they *really, really, really* are intrigued by watching mud form when it rains. They have not seen this before! So they add visiting a dirty field during a rainstorm. All 3 of them vote for that while 2 vote for the chocolate factory, 2 for the art museum, 2 for the airport, 1 for American Tobacco, 2 for UNC, and 1 for the farmers' market. What do you think?

Rank Choice and Runoff Voting

What can we do to make sure that a small minority don't force their very different views on everybody else? I bet none of the other students would at all want to go to the field. In small groups explore the idea of ranking – not voting for one but maybe ranking all of the choices. I'd love to go to the art museum as my first choice, chocolate factory as my second choice, farmers' market as my third choice, and am okay with the other choices except for the field, which I rank last.

Or we do a runoff. If there is no simple majority, we get rid of the lowest vote getting choices and vote again. We do this until we get a majority.

Take a few minutes to consider these ideas. In small groups vote on something with three or four choices, maybe colors to paint your school. See how you can make a decision with rank choice and then by runoff voting. While exploring, consider questions like:

- If you have 4 choices and have definite preferences, you could rank your top choice 4, next choice 3, next choice 2, and last choice 1. Can we add all the "scores" and decide a winner? What do you think of this system?
- What if I love one choice, hate one choice, and don't care about the other two. What kind of ranking system could you use?
- Now consider a runoff. What rules would you use to eliminate some choices?
- How would you do a runoff – ask folks to vote again? Or would you do an "instant runoff" and find each choice's score based on the preference ranking numbers?
- Can a choice be a "spoiler"? What if we're voting on fruits – I love mango and like peach but don't like dragonfruit. Not many others know mango. Should I vote on peach to make sure that I have a good choice and give up on what I really want? If peach weren't a choice and mango might win, does that make peach become a "spoiler"?

Take your time – there's a lot to consider! Try voting on things like colors of your school, a fruit to buy for a party, a book to read, or something else. We might even try doing some of this with the whole group.

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Voting can be complicated – but is the bedrock of democracy. I hope that if your family is eligible that they vote now and in all future elections. Voting systems rely on people who understand how to use tools like math to help make voting as reasonable as possible.

Have fun – keep on thinking about voting and how to make choices in a group to make the most people happy!

Enjoy
Mr. Barman

Notes for Parents

When I was thinking what I would present, I had some topics that I've already developed in other contexts that I could easily have put together for our late October session. But it kept nagging me – it seems everybody, even younger children, know that it's election season and that big ideas and many candidates are in the running to try to capture citizens' favor. How could I not do a session about voting?

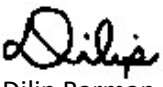
I thought that I would make today's session one of thinking about some of the issues about voting. The lesson was very loosely inspired by a lesson from Illustrative Mathematics: Learn Math for Life, "More than Two Choices"[2] from an overall 6th grade lesson on voting. The mathematics of voting is a rich area of study; in homeschool I covered this in some detail with my daughter in 6th grade, as a matter of fact.

If this were a math circle for middle school, we could have gone into more details, but I think that today's session will be a valuable one to plant seeds that elections are complicated and the kinds of analytic skills that these children are developing are important in yet another area of life. I hope that the activities and reflections will help students to start to see how voting is generally much more complicated than picking from two excellent choices, readily getting a winner, and everybody being happy. We can do certain things – encourage multiple choices, structure the vote so that spoilers have minimal impact, think about key fairness and happiness criteria – analytically to help the voting process help groups that we are part of make decisions.

Of course there are factors beyond math such as combating apathy, bringing people together, and seeking commonality and compromise for the greater good, that are also important in running a democratic institution. Analysis can also be used for the opposite purpose to further one's particular goals by techniques such as gerrymandering. But hopefully the peek that the children will get today will be a positive one and help them put a little context into this election season.

To go into more detail on the richness of electoral mathematics, some good references for adults, as well as middle school and older students, are the article with included video, "Elections: Could they be fairer" by Chris Budd[3], "Math 110: An Introduction to the Mathematics of Voting"[4], and "Can Voting Ever Really Be 'Fair'" by Linda Marie Saliga and Lynne M. Pachnowski[5] from a math circle journal. You may want to introduce elements of some of these with your younger children.

Till next time!



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