Chapel Hill Math Circle
Session 5 – November 9, 2024:
"Bring on the Cranberries": Mathematical Puzzles and Games of Food
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com

Supplies needed: 30 cranberries (or tokens) per student pair, paper, colored pencils

Welcome to Math Circle session 5! Thanksgiving is coming up – did you know that I host the country's largest vegetarian (all vegan) Thanksgiving? Please join in on Thanksgiving Day or the Monday before Thanksgiving for a pre-Thanksgiving. Your parents can get details at trianglevegsociety.org/thanksgiving. We're on most of the evening news programs most years!

Let's use this opportunity to do some fun mathematical food puzzles. But first … where do you think math plays a role in food?

"The Four Days of Thanksgiving"

Let's make up a song inspired by *The Twelve Days of Christmas* called *The Four Days of Thanksgiving*. Maybe it goes something like this:

- On the first day of Thanksgiving my Lovely Chef gave to me 1 artichoke salad.
- On the second day of Thanksgiving my Lovely Chef gave to me 2 mushroom roasts and an artichoke salad.
- On the third day of Thanksgiving my Lovely Chef gave to me 3 shepherd's pies, 2 mushroom roasts, and an artichoke salad.
- On the fourth day of Thanksgiving my Lovely Chef gave to me 4 cherry tarts, 3 shepherd's pies, 2 mushroom roasts, and an artichoke salad.

How many gifts are given on the 1st day? The 2nd day? The 3rd day? The 4th day? In total?

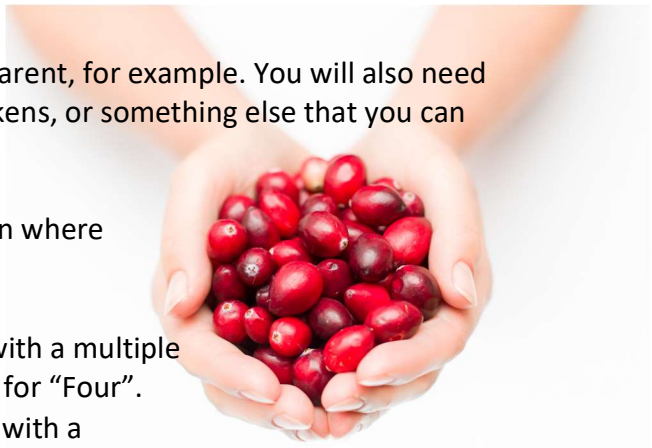
Do you remember Pascal's Triangle?

Pascal's Triangle was first discovered in ancient China. It consists of rows of numbers – a 4-row one is here to the right. Can you figure out its pattern and add on at least 3 or 4 more rows?

1
1 1
1 2 1
1 3 3 1

See if you can find the number of gifts in Pascal's Triangle. Look for the daily gifts and the total gifts.

Let's play the "cranberry game"[1]



You will need a partner; it could be another student or your parent, for example. You will also need 30 "cranberries", which could be actual cranberries, coins, tokens, or something else that you can easily handle.

The goal of the game is to force your opponent into a situation where they can't take any cranberries. Here are the rules:

- One person is the "4" person; he or she must end up with a multiple of 4 (0, 4, 8, …) after each turn. Let's call that person F for "Four".
- The other person is the "6" person whoe must end up with a multiple of 6 (0, 6, 12, …) after each turn. Let's call him/her X for "siX".
- Put 30 cranberries in the middle in the "field".

[1] picture "Cranberries" by Christian Dembowski, taken on Nov. 13, 2013, from flickr.com/photos/christiandembowski/10843772895, accessed Nov. 8, 2024; freely usable under CC - Attribution-Noncommercial-No Derivative Works 3.0 license creativecommons.org/licenses/by-nc-nd/3.0/

- Decide who starts; players take turns. On each turn a player must end up with the appropriate multiple of cranberries. They can take any number (including 0) up to 10 from the field. Then they can optionally trade cranberries 4-for-3 with their partner.

For example, here is what the first two turns could look like where we keep track of (cranberries in the field, cranberries that F has, cranberries that X has), starting with (30, 0, 0):

- (30, 0, 0); F starts and takes 8 to end up with (30-4, 0+4, 0) = (26, 4, 0).
- X takes 9 to get (17, 4, 9). 9 isn't a multiple of 6 so X gives 4 to F to get 3: (17, 8, 12).

Play a few rounds. Do you have a strategy to win?

How many salads[2]?

Let's visit a salad bar.

- Maybe we're early – there's nothing to select from. We have **0 ingredients** and only **1 choice** – an empty plate.
- Now we have one choice, avocadoes. With **1 ingredient** we can have an empty plate (no avocado for me, please) or a plate with avocado; **2 choices**.
- With 2 ingredients we can pick 0 ingredients (empty plate), 1 ingredient (avocado or banana), or both ingredients; **2 ingredients** gives us 1+2+1 = **4 choices**.
- How many choices of plates do we have when there are 3 ingredients? We could choose 0 ingredients 1 way – an empty plate. We could pick a plate with just 1 ingredient – avocado only, banana only, or cherry only. We could have 2 ingredients – avocado and banana, avocado and cherry, or banana and cherry. Finally there's 1 way to have all 3 ingredients. So with **3 ingredients** we have 1 + 3 + 3 + 1 = **8 choices.**
- Keep going – make a table. How can Pascal's Triangle help us?

Number of ingredients	Number of choices
0	1
1	1+1 = 2
2	1+2+1 = 4
3	1+3+3+1 = 8
4	
5	
6	

Enjoy
Mr. Barman

[2] Picture by Viji Sathy, July 20, 2012, of some food that I had catered for a party. That's my now-almost-16-year-old daughter with me.

Notes for Parents

On Thursday night, November 7, 2024, I sat down to plan today's math circle topic. I was thinking of a part 2 for the voting lesson (there's so much math in elections – but … no) or maybe a lesson on games (what is a "winnable" game and considering some extensions of well-known games). But then I thought of food!

I hope that the students will have a fun time with the activities here. Each of them has open-ended extensions that you can explore with your child if you wish.

For the "Four Days of Thanksgiving", go on to larger and larger numbers of days. Do you know about Triangular Numbers (0, 1, 1+2=3, 1+2+3 = 6, 1+2+3+4 = 10, …)? How do they relate to the daily gift totals? If you can find these numbers in Pascal's Triangle, can you find nearby the totals of all the days?

I bring up Pascal's Triangle in various math circles periodically. There are lots of patterns in it! You might look with your child to see how many you can find. What do you get if you add each row? How about squish together the numbers on each row? What do you see on the diagonals? If you remember binomial expansion, do you see the binomial coefficients in the Triangle?

I teach lots of math classes (please look into my classes in-person at Mathematics Institute of the Triangle – trianglemathinstitute.com) but one thing I love about math circles is that sometimes I put together questions for which I don't know the answer. It's fun exploring with the students. That's the case with my cranberry game. I am not sure what the strategy should be or even if it will prove to be a particularly worthwhile game. It came to me tonight and I thought it may be an interesting activity. You can extend this with your child by changing the parameters – the total number of cranberries, the rules of exchange, and the multiples. You could use this to reinforce times tables with multiples and start presaging the concept of least common multiple.

It's so interesting to consider "salad bar numbers". Those numbers are just the binomial coefficients that can be found in Pascal's Triangle! I've had math circle sessions where we've put significant focus on this. Try taking your child to an area salad bar; I love the ambiance at Weaver Street Market. Have them count the number of choices when you have them look at just a few ingredient choices. We probably won't have time, but there is a cool pattern they may be able to see – each additional ingredient doubles the number of choices. It makes sense – we have the previous choices with and without the new ingredient.

Till next time!

Dilip

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