Chapel Hill Math Circle
Session 6 – November 23, 2024:
Fibonacci / Pingala Day
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com

Supplies needed: graph paper, colored and regular pencils optional compass

Welcome to Math Circle session 6! Thanksgiving is around the corner and last time we did some math and food exploration. Today is Fibonacci Day (but it should be called Pingala[1] Day) so let's celebrate!

Let's dive in and identify the Fibonacci pattern

Have you heard of Fibonacci Numbers? Let's generate them. Write down 0 and then 1. That's how the sequence begins, though it can also begin with two 1s. Now use a rule: take the last two numbers, add them up, and get the next number. Try it and write the first few such numbers – called Fibonacci Numbers.

A little bit of history

These numbers were known to ancient Chinese and Indian mathematicians but were made famous about 1400 years later by Leonardo Bonacci, known as Fibonacci, a mathematician from Pisa in modern day Italy. He wrote a famous book, *Liber Abaci*, and in it asked how many pairs of rabbits[2] we have over time if babies take 2 months to grow old enough to themselves have babies. Once mature, rabbits have a new pair of babies each month "forever". He also introduced the idea of zero to Europe in this book.

Historically it seems that Indian poet and mathematician Acharya Pingala first came up with these numbers around 200 BC. He used it to describe Sanskrit (perhaps the world's oldest language and the root of many Indian languages) poetry's syllables. In his book *Chandahsastra* he wrote about topics like these

[1] This sculpture picture of Pingala is from booksfact.com/science/ancient-science/pingala-inventor-of-binary-numbers-in-2nd-century-bce.html, accessed Nov. 15, 2022.
[2] The picture is from a 1958 book called *The Rabbits' Wedding* by Garth Williams. This image is from vintagechildrensbooksmykidloves.com/2008/09/rabbits-wedding.html, accessed Nov. 18, 2024.

"Fibonacci" numbers as well as other topics like "binary numbers" (very important in modern technology) and a topic we have looked at and will continue to consider, Pascal's Triangle, again named after a mathematician who lived over 1000 years after the original invention.

Let's look for patterns in dates

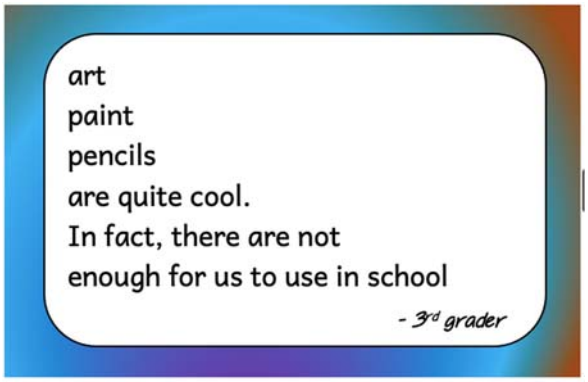
We have 12 months and often abbreviate them. For example, July 4, 2024 might be written as 7-4-2024; the 4th day of the 7th month in 2024. Sometimes we abbreviate further and just say 7-4-24 if it's clear it's a recent date. If we want to refer to the U.S. Independence Day, we'd say 7-4-1776.

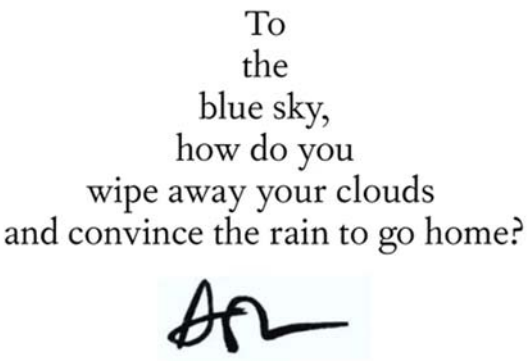
Just to be sure that you are comfortable with the code, write down each of the twelve months and the number associated with each. Then practice writing shortcuts for various dates of your choosing, such as maybe the day you were born or, say, the 18th day of October.

Does this give you a hint as to why November 23 each year is called Fibonacci Day?

Fibonacci Poems

In a Fibonacci Poem the first line is 1 syllable long, the second also 1 syllable, the third 2 syllables, the next 3, then 5, and so forth. Alternatively, words instead of syllables can be counted. Here are a few that I found[3].



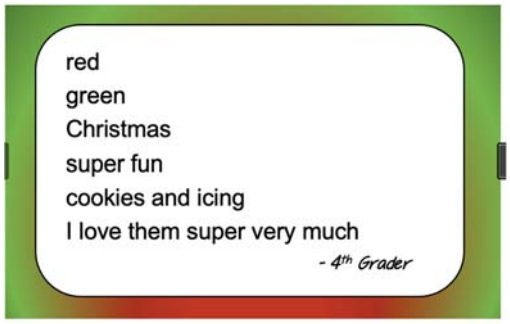


[3] The sky one is by Ash Raymond James from January 2021 found at ashraymondjames.com/p/the-truth-about-fibs, accessed Nov. 28, 2022; the ones with colored borders are from bigideas4littlescholars.com/fibbing-can-be-fun-poetry-gallery, accessed Nov. 28, 2022; and the longest one is from Brian Bilston found at brianbilston.com/tag/fibonacci-sequence, accessed Nov. 28, 2022.

I
wrote
a poem
on a page
but then each line grew
by the word sum of the previous two
until I started to worry at all these words coming with such frequency
because, as you can see, it can be easy to run out of space when a poem gets all Fibonacci sequency.

What about going forwards and then "inverting" and going backwards? Here is a poem by Prem Nizar Hameed[4] about one of my favorite fruits, the mango.

Try writing your own poems! Make it a Fibonacci one or an inverted one. Bend the rules a bit if you want!



National Mango Day

One
pinch
of salt,
some piquant
red chilli powder,
a spoonful of small onion paste,
a small quantity of coconut oil, and apply
the mixture on the raw mango pieces thoroughly;
then have a piece in slow motion

to make nostalgic
national
mango
day
now

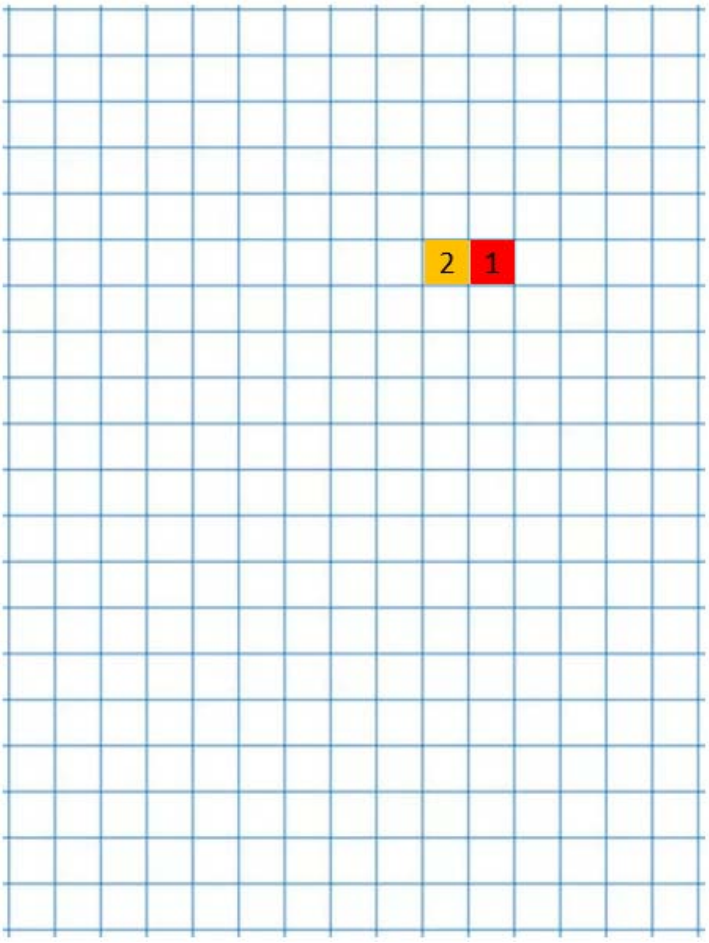
(A Fibonacci inverted poem; Syllable: 1-1-2-3-5-8-13/13-8-5-3-2-1-1)

Prem Nizar Hameed

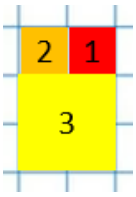
[4] from poemhunter.com/poem/national-mango-day, accessed Nov. 29, 2022

Fibonacci Rectangles

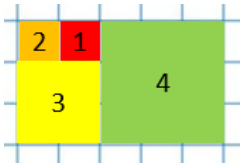
Let's make Fibonacci Rectangles. Use graph paper and colored pencils. Start with coloring in 2 squares next to each other different colors. Each square is 1 square across. Try putting the squares about ¼ down and about 2/3 across, roughly like this to the right (you don't have to number your squares; I have numbers just to guide you).



The Fibonacci Sequence starts with two 1s – and you have two squares of side 1 (we are measuring on the graph paper grid lines). Let's now make a square in another color with each side 2. Put that just below the squares you have so far to make a rectangle which is 3 units tall and 2 units wide like to the left.



Okay, the sequence is 1, 1, 2, 3; what do you think will happen next? Did you get something like this?

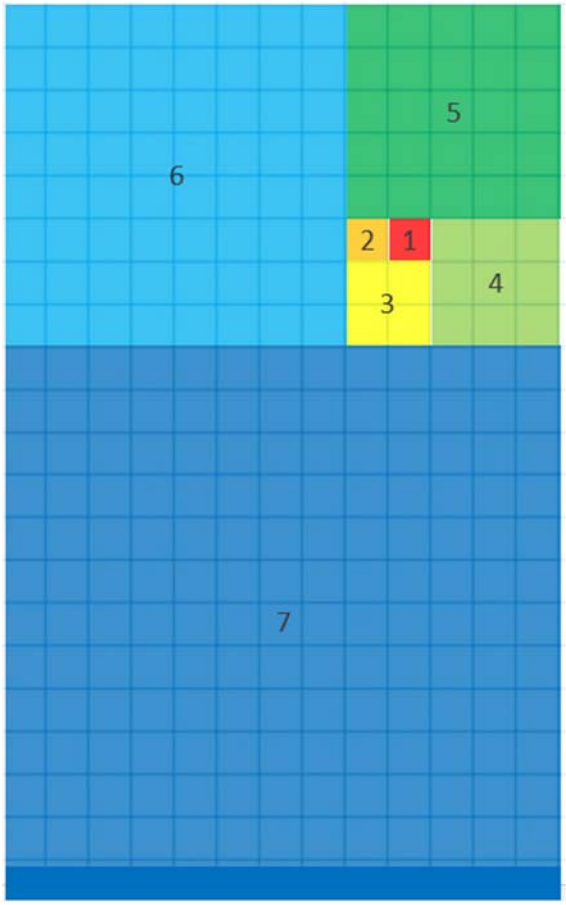


Keep going for a few minutes or until you run out of space on your piece of graph paper. On the next page is an example of what you might come up with.

What Patterns do you See?

Besides the beauty, do you see any patterns? We started with a square that is 1 tall and 1 wide; since all squares are also rectangles, we call this a 1x1 rectangle. We add the orange square to get a rectangle that is 1 tall and 2 wide - a 1x2 rectangle. We can also call this 2 wide x 1 tall; let's agree to give the shorter dimension first. Keep going; what happens to your rectangle at each step? Please take a minute to think about this.

Did you get something like this with Fibonacci Rectangles?



Did your Fibonacci Rectangles look something like this to the right? I've shown the grid lines to make it easier to count and look for patterns.

We start with a 1x1 rectangle; we add the orange square to get a 1x2 rectangle. Then we add the yellow 2x2 square to form a 2x3 rectangle. (remember that this rectangle is 3 tall and 2 high, but if we turn it on its side it would be called a 2x3 rectangle and we decided to always give the shorter length first).

Okay, adding the light green square makes our rectangle be 3x5; next with dark green we have 5x8. I think I see a pattern! When we next add an 8x8 light blue square we end up with a rectangle that is 8x13. Finally we add the big dark blue 13x13 square and get a big rectangle that is 13x21.

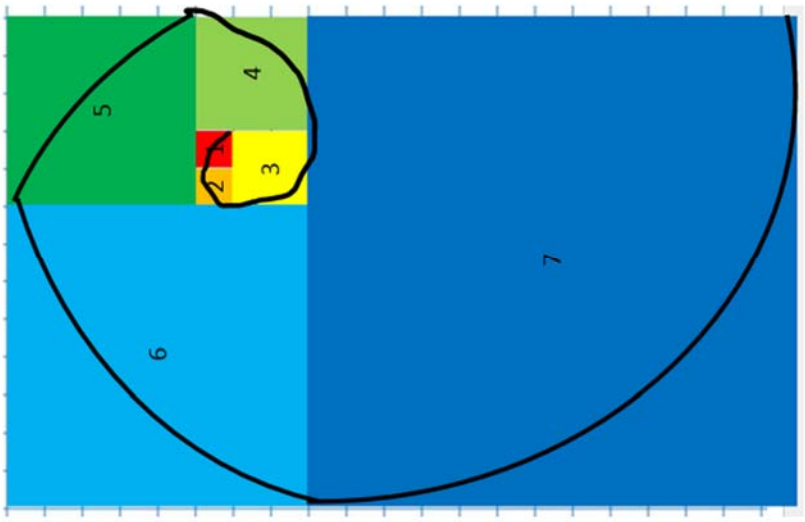
Let's organize our information into a list:

- 1x1
- 1x2
- 2x3
- 3x5
- 5x8
- 8x13
- 13x21

Do you see the Fibonacci Numbers – twice over?!

If we pick the smaller dimension of each rectangle, we get … the Fibonacci Numbers 1, 1, 2, 3, 5, … ! If we pick the larger dimension, we get … basically the same thing (though the first 1 is missing)!

By the way, freehand or with a compass, you can make a Fibonacci Spiral. It should look something like the one I drew here turned to its side so it will fit.



Rabbits

We probably won't have time but if we don't, here are some problems for you to consider on your own. Let's think about rabbits[5].

Fibonacci wanted to use math to model just how many rabbits there might be starting with one pair. Let's say that we start with one pair of baby rabbits. Here are some simplified assumptions that he made:

- Baby rabbits and 1-month old rabbits are too young to get married
- The rabbits get married and have a pair of rabbit babies as soon as they are 2 months old – and every month after that
- A pair of baby rabbits is what we have to start; no rabbits enter or leave the area

As math lovers we use math and model the world with simplifications. These are clearly unrealistic expectations – rabbits don't live forever, rabbits may die or leave the warren, new rabbits may join the warren, brothers and sisters don't get married, and more. But let's go with these as starting assumptions to get a basic model going that we can tweak. Let's run the model.

Month 0: We start with 2 baby rabbits. They are destined to marry and have two more babies. So we have 1 pair. Let's call them Adam and Aileen.

Month 1: The pair is 1 month old so can't get married. We still have 1 pair waiting to get married.

Month 2: Congratulations! Adam and Aileen are married and have babies – let's call them Ben and Benita. We now have 2 pairs of rabbits.

Month 3: Adam and Aileen have their second pair of babies (maybe called Carl and Connie), so we add a pair to our count and have 3 pairs. Ben and Benita are hoping next month to get married!

Month 4: You guessed it, Adam and Aileen have another pair of babies – add 1 to get 4 pairs! But so do newlyweds Ben and Benita so we have 5 pairs! Carl and Connie are dreaming of getting married. We continue in this way and get the Fibonacci sequence: 1, 1, 2, 3, 5, … !

Consider this in the sense of population dynamics. In a country like Japan where there is an aging population and not enough people to fill jobs, should a country have policies to encourage children? How about a country like India which has a very large population?

Enjoy
Mr. Barman

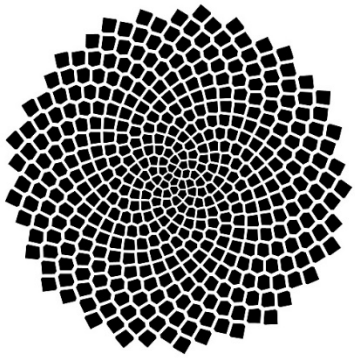
[5] picture from omlet.us/guide/rabbits/should_i_get_rabbits/wild_rabbits, accessed Nov. 14, 2022

Notes for parents

Hello! We often look to nature for rejuvenation – lovely autumn hikes, winter skiing, spring romance with budding flowers, summer lake visits, and much more. Nature is a great source to which to turn with our children as well in exploring mathematics.

Why do we find math in nature? What is the connection? Is our maker - mother nature, God, whatever power or force in which we believe – concerned with mathematics? Do we use math to explain nature? All of the above!

Patterns are fundamental to math and I like in our math circle to dedicate a lot of time to explicitly looking at patterns – implicitly we are almost always doing that in math as well. Why is it that so many plants follow Fibonacci numbers in their structure?



A great example is the sunflower. Seeds within the sunflower form fascinating patterns as I included in my images from MoMath's website[6] (and I continue to heartily recommend the National Museum of Mathematics as a great resource with many online programs). If we look carefully we find 21, 34, or 55 spirals of seeds – each of these is a Fibonacci number!

Why do these numbers show up so many places? One reason is for efficient packing. Informally, using 3rd or 4th grade math and division, if we start at the center and spread out lines of seeds, we get very inefficient packing if we use fractions – e.g., every quarter turn and we have four lines with a lot of wasted space with no seeds. But if we push seeds out and turn an irrational fraction, we get more efficiently tightly packed seeds. What is the most irrational of irrational numbers? It isn't Pi but is Phi. Phi is approximated by dividing consecutive Fibonacci numbers!

I love a Numberphile video[7] on this topic that I highly recommend for you and for 5th graders and up. There is a cool Math is Fun interactive page[8] that you might enjoy having your child tinker with.

If I have time I'll introduce today the idea of recursion in defining Fibonacci Numbers. We can use self-referential recursion to say that the nth Fibonacci number is the sum of the last two. We need a "stopping rule" so define the first one. In mathematical notation, we have:

[6] momath.org/home/fibonacci-numbers-of-sunflower-seed-spirals, accessed Nov. 15, 2022
[7] youtube.com/watch?v=sj8Sg8qnjOg&list=PLt5AfwLFPxWKMXtxxL5qm9AcarCzNJDM0&index=1
[8] mathsisfun.com/numbers/nature-golden-ratio-fibonacci.html

$$\forall n \in Z, Z > 1, F_n = F_{n-2} + F_{n-1}$$

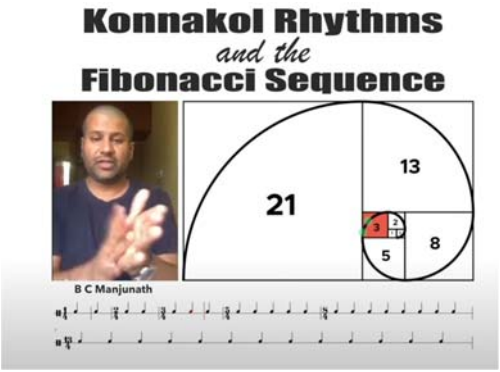
$$F_0 \equiv 0,\ F_1 \equiv 1$$

Recursion is a powerful technique that is a bit counter-intuitive – we don't define in language arts things with respect to themselves. If somebody from Mars visited and asked what a colored pencil is, we wouldn't say something like, "it is a pencil that writes in colors other than necessarily black", but rather something like, "it is a writing instrument, often made with wood and filled with a substance that, when rubbed on paper, leaves a mark; a colored pencil is one which leaves a mark of a variety of wavelengths of light".

There is a lot to say about Fibonacci numbers and beauty in nature and many other domains. I have a long document of dozens of pages on this topic that I created in homeschool for my daughter when she was in 5th or 6th grade.

Here are some activities for your child that you might consider:

- Have your child list more Fibonacci Numbers to give them good addition practice.
- Have your child figure out how many rabbits there are by this model month-by-month.
- What if you change the model so that it takes longer for rabbits to mature? Perhaps you can explore in a simple way some probabilistic models where only a certain percent of rabbits have babies each month. You can use these exercises to discuss mathematical modeling.
- Fibonacci Numbers can be found in Pascal's Triangle! Squish all the numbers to the left to make a right triangle – can you see them? Look in the diagonals and add some numbers.
- Gift your child with a sunflower and have them find Fibonacci numbers.
- Look at a variety of flowers and keep track of the number of petals; how many of them are Fibonacci numbers?
- Go for a hike and look for pinecones – does your child see Fibonacci numbers?
- Pick up a pineapple – are there Fibonacci Numbers?
- Try using a compass to make a smooth Fibonacci Spiral.
- Keep making Fibonacci Poems.



- Fibonacci sequences are used in Classical Indian music and other types of music too. Check out a fun video[9] showing Carnatic musician B. C. Manjunath clapping rhythms while visualizing the Fibonacci Spiral[10].

Happy Pingala / Fibonacci Day! Happy Thanksgiving!

Till next time!

Dilip

[9] youtube.com/watch?v=Zc7WMbVW-3s

[10] This – as well as the picture here - is from a National Public Radio story available at npr.org/2018/08/10/637470699/let-this-percussionist-blow-your-mind-with-the-fibonacci-sequence. Check out as well the rhythms at youtube.com/watch?v=PSU-4ZJ_fJ0.