Chapel Hill Math Circle
Session 9 – February 1, 2025:

Big Numbers

Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com

Supplies needed per student: many stackable objects – ideally a few hundred pennies or thin stackable wooden tiles, as well as paper and pencil

Welcome to Math Circle session 9. Happy February! Numbers are cool as they go on forever and forever. Let's spend an hour thinking about big numbers.

What is a big number?

If you want to think about big numbers, doesn't "big" depend on what you're talking about? Take a few minutes and write down what "big" means in different contexts. What is a big score? A big distance? What does big mean to a gardener? To an astronomer? Jot down some ideas.

It's impractical to write some really big numbers so we use scientific notation

When we say that the sun is 93 million miles away, is it exactly 93,000,000 miles away? Think about how precise our instruments let us be and what distance means. Is it from the top of a mountain to the center of the Sun? Is it from a valley? Does it really matter?

When we deal with really big (or really small) numbers, we sometimes use "order of magnitude". We use a small number next to 10 called a "superscript" to tell us how many zeros to tack on to the number. Check out the chart on the next page[1]; we will also see a cool video called "The Cosmic Eye"[2] by Danail Obreschkow. I won't show the whole thing, but let's take a peek at the beginning of a 9-minute video "Powers of Ten"[3] by Charles and Ray Eames of IBM.

[1] page 157 of the book *Organizing Science Information and Concepts*, accessed Jan. 31, 2025 from csun.edu/science/ref/measurement/orders_of_mag/sourcebook-8.2.pdf
[2] youtube.com/watch?v=8Are9dDbW24; we also saw this in a math circle session in October 2024
[3] youtube.com/watch?v=0fKBhvDjuy0

Distance	Comparison (Approximate)
10^{0} m	Distance from floor to door knob
10^{-1} m	Width of hand
10^{-2} m	Width of fingernail on smallest finger
10^{-3} m	Thickness of a U.S. dime
10^{-4} m	Length of a dust mite
10^{-5} m	Diameter of human red blood cells
10^{-6} m	Diameter of small bacteria
10^{-7} m	Length of a virus
10^{-8} m	Thickness of bacteria flagellum
10^{-9} m	Width of DNA helix
10^{-10} m	Width of ice or quartz cell
10^{-11} m	Radius of a hydrogen atom
10^{-12} m	Wavelength of X-rays
10^{-13} m	Wavelength of an electron
10^{-14} m	Diameter of a nucleus
10^{-15} m	Diameter of a proton
10^{-16} m	One-tenth the diameter of a proton
10^{-17} m	One-hundredth the diameter of a proton
10^{-18} m	Radius of an electron

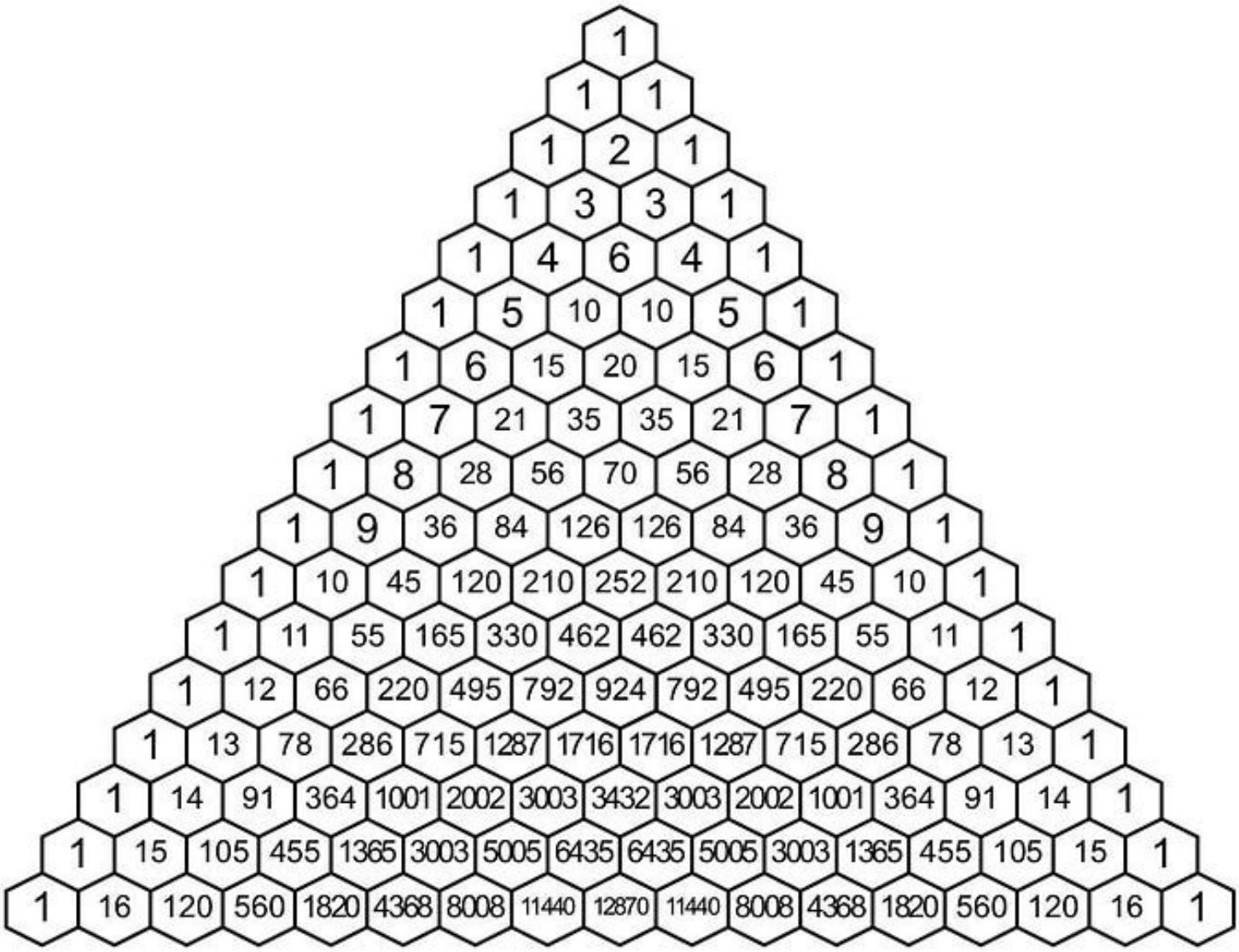
Increasing Size →

Distance	Comparison (Approximate)
10^{26} m	Radius of observable universe
10^{25} m	Distance to the 3C273, brightest quasar
10^{24} m	Distance to the nearest large supercluster
10^{23} m	Distance to galaxies beyond our local group
10^{22} m	Distance to Andromeda galaxy
10^{21} m	Diameter of the disc of the Milky Way
10^{20} m	Diameter of the Small Magellanic Cloud
10^{19} m	Approximate thickness of the Milky Way
10^{18} m	Diameter of a typical globular cluster
10^{17} m	Distance from Earth to Vega
10^{16} m	Inner radius of Oort cloud
10^{15} m	100 × diameter of the solar system
10^{14} m	10 × diameter of the solar system
10^{13} m	Diameter of solar system
10^{12} m	Distance from Sun to Saturn
10^{11} m	Distance from Sun to Venus
10^{10} m	One half the distance light travels in a minute
10^{9} m	Diameter of the Sun
10^{8} m	Diameter of Saturn
10^{7} m	North Pole to equator
10^{6} m	Length of California (north to south)
10^{5} m	Length of Connecticut (north to south)
10^{4} m	Depth of Mariana Trench, deepest point
10^{3} m	One kilometer; 2.5 times around a track
10^{2} m	One side of a running track
10^{1} m	Distance for a first down in football
10^{0} m	Distance from floor to door knob

Showing how big a number is

We take for granted using numbers that we consider to be “big”. Do you remember Pascal’s Triangle? Take a minute to make one to at least 5 rows.

Here is a 17-row one[4]! Are you surprised how large the numbers get?



On the next page is a picture of one of my Stanford Math Circle 1st grade students, shared with his and his dad's permission, from January 2025. He took 2 ½ hours to make this huge 11-row Pascal's Triangle!

[4] from cantorsparadise.com/8-secrets-of-pascals-triangle-349cb5e46b09, accessed Jan. 21, 2025

Let's try "making" big numbers!

Let's use the rest of our time to make big numbers. You can do as many of these activities as you would like and have time for.

- Make a Pascal's Triangle out of pennies, blocks, or anything else stackable. Be sure to have several hundred of the items! How many rows can you build?
- Have you heard of the ancient Indian story about a chessboard? One grain of rice is put on the first of the 64 squares of the board and it is doubled every day. You get an astounding number of grains of rice – in fact, more than there are grains of rice in the world! Try this; stack an object, then stack double that object, and continue. See if you can repeat this 5 or 6 or more times. You practically won't be able to double 10 times – can you guess how many would be in the stack if you did?

- Have you heard of factorials? 4! is 4 groups of 3 groups of 2 groups of 1; 2 groups of 1 is 2, 3 groups of 2 is 6, and 4 groups of (3 groups of 2 groups of 1) is 24. Experiment with your objects. You'll be surprised how large factorials grow. See if you can figure out what 5! and 6! is. Does this chart[5] (don't look till you try finding 5 and 6 factorial) surprise you?!

	N	Factorial
1	1	1
2	2	2
3	3	6
4	4	24
5	5	120
6	6	720
7	7	5040
8	8	40320
9	9	362880
10	10	3628800
11	11	39916800
12	12	479001600
13	13	6227020800
14	14	87178291200
15	15	1307674368000
16	16	20922789888000
17	17	355687428096000
18	18	6402373705728000
19	19	121645100408832000
20	20	2432902008176640000

More to think about

When we think of big numbers, do all the digits matter? Would you have different expectations of a city with population 1,023,111 as opposed to one of population 1,000,000? How can we even be sure how precise such numbers are? Are there other ways to literally show how big some numbers are?

Have Fun!
Mr. Barman

[5] encyclopedist-dictionary.blogspot.com/2013/07/factorial.html, accessed on April 23, 2018

Notes for Parents

When we teach our children numbers before they begin school, the idea of infinity sometimes comes up. One of the beauties of math is the ability to imagine something that is impossible to see or enact – like a population of 6 million. We can't even count to 6 million in a lifetime!

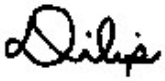
While abstraction is important in math, it is good to also be grounded. It's easy to make a Pascal's Triangle and use bigger and bigger numbers, but do we really understand the magnitude of those numbers? I was so proud of my Stanford Math Circle student pictured in today's lesson with his physical Pascal's Triangle (train station and all leading up to the towering Triangle!) and delighted that his dad and he were happy for me to share it (see as well https://bsky.app/profile/dbarman.bsky.social/post/3lgg6gn2rkk2s). I hadn't thought of making the Triangle concrete like that. The blocks quickly get taller than the student!

I recommend that you continue to work with your child in numeracy. When shopping estimate the number of tangerines in a case, for example – and then count to see how far off your estimate is. Talk about numbers that are difficult to experience. Guess the number of dust particles that might be in barn or the time it would take to get to a planet.

One thing I wish were more covered in schools is estimation. Students do learn about concepts like precision vs. accuracy in high school chemistry but before that do they really think about considering what it means if a weigh scale says that a person is 62 pounds vs. 62.0 pounds?

I hope that the foray into big numbers was fun. The two videos are excellent and well worth seeing again.

Till next time!



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P.S. Did you sign up for the Math Kangaroo test? Late registration ended yesterday but I wonder if it might still be possible to register. Check out mathkangaroo.org. My school, Mathematics Institute of the Triangle is one of the many schools hosting it around the world.