Chapel Hill Math Circle
Session 2 – September 13, 2025: Polygons and Diagonals
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com
Supplies needed: rulers, spaghetti, paper, colored pencils, optional rope

Welcome to Chapel Hill Math Circle! We're glad that you are here to have some fun! If you were here last time, we talked about shapes. Let's review a bit.

We discussed **closed shapes** where something could be contained. **Polygons** are closed shapes with straight lines. A **simple shape** has no crossing lines. **Regular polygons** have all the lines the same and all of the angles in between the lines the same.

Let's focus on simple polygons today and discuss convex and concave polygons, diagonals, and maybe triangles today. Ready to have fun?

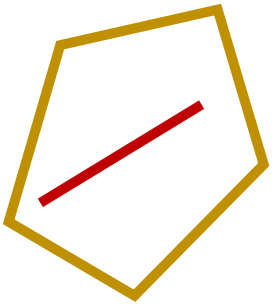
Convex and Concave Polygons

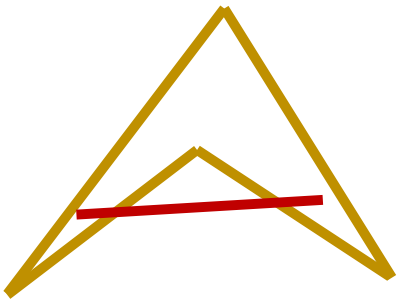
Can you draw some simple polygons where if you travel in a straight line from one part of the polygon to another you always stay inside of the shape? These are called **convex**. Now make some simple polygons where a straight line leaves the polygon. These are called **concave**.

Concave	Convex

I like to remember this by thinking of a concave shape as having a "cave". You can also think about this as taking a straight march; do I have to show my passport to enter another country and then re-enter my country? If so then my country's shape is concave.

Here are examples I made of concave and convex shapes. No matter where I draw a line in a convex shape, I always stay inside the shape. That's not true for a concave shape! Make a few more examples of concave and convex shapes below.





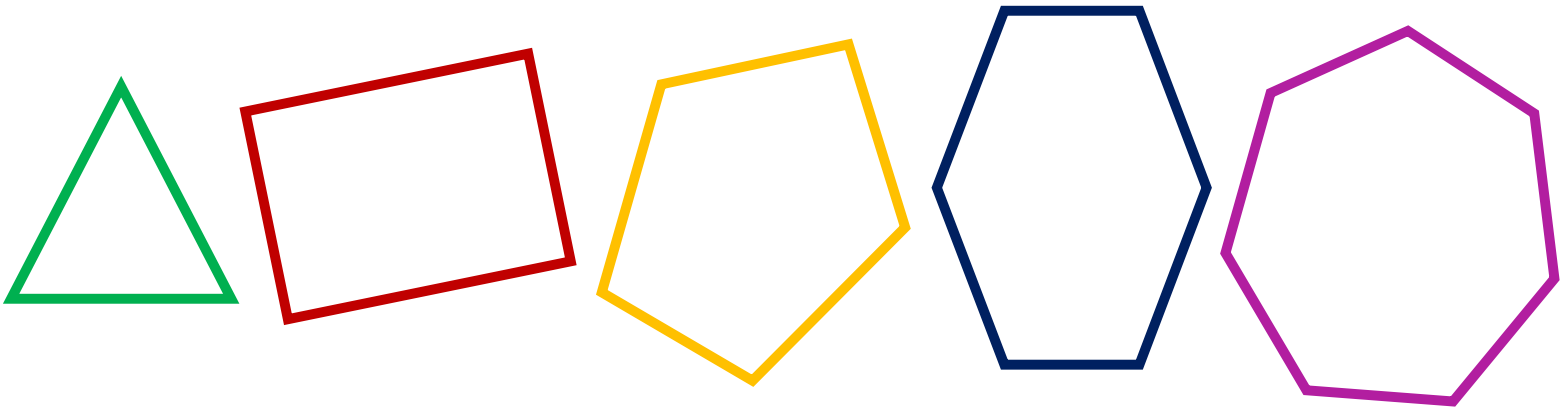
Let's get up and move around - can you work with a partner to make a shape with rope that is convex? Concave? How about with your body? Can you move a piece of paper or index card to prove that a shape is convex or concave?

Concave	Convex

Diagonals

What is a diagonal?

What do you think a diagonal is? Can you use different colors to draw all the diagonals in these polygons? Do all triangles have the same number of diagonals? All 4-sided polygons? All 5-sided ones? All any-sided ones?

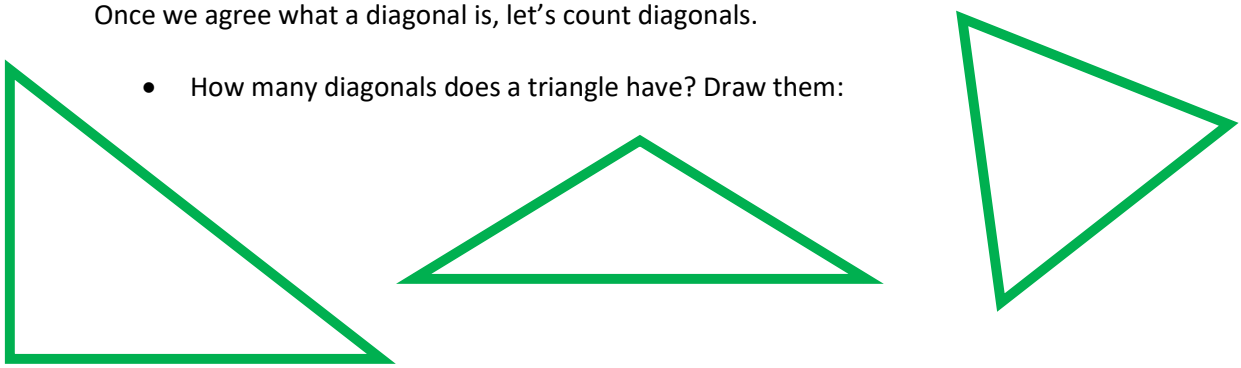


(Don't peek till I tell you: A diagonal is defined as a line segment between two "vertices" of a polygon that aren't directly connected.)

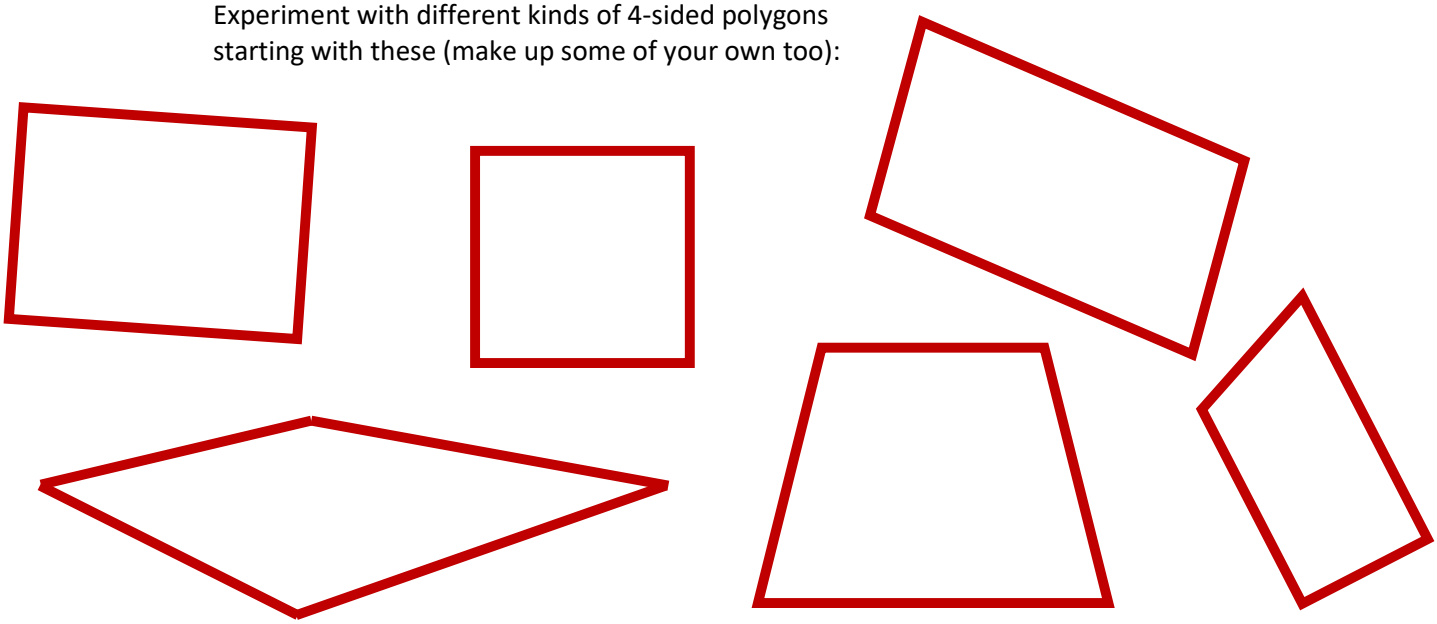
How many diagonals?

Once we agree what a diagonal is, let's count diagonals.

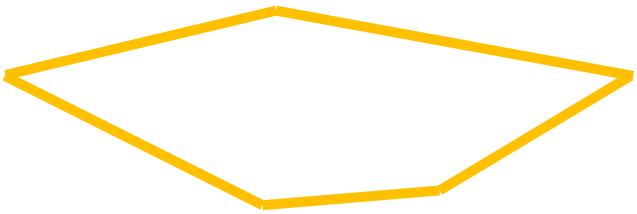
- How many diagonals does a triangle have? Draw them:

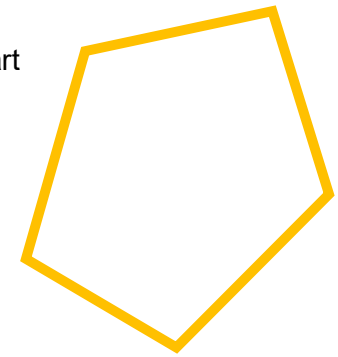


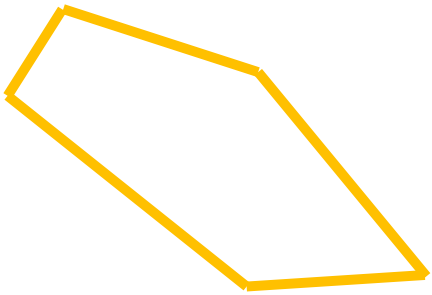
- How many diagonals do 4-sided polygons have? Experiment with different kinds of 4-sided polygons starting with these (make up some of your own too):



- How many diagonals do 5-sided polygons have? Experiment; start with these:





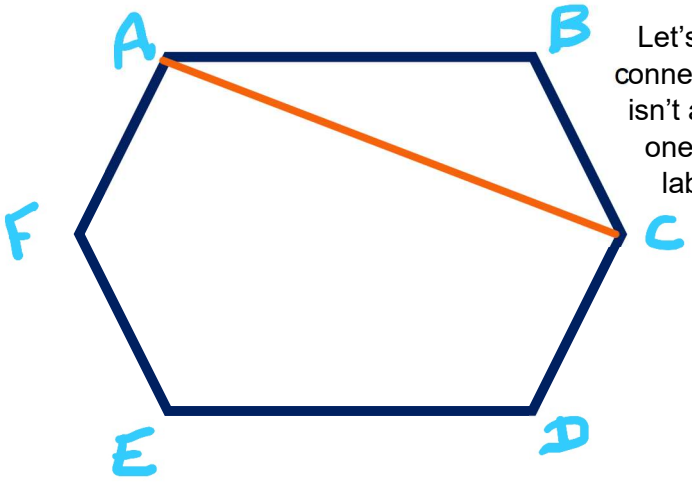


Make a table of what you have found so far. Do all polygons of the same number of sides have the same number of diagonals? If not then list all the possibilities.

Do you have a prediction of how many diagonals a 6-sided polygon would have? How about 7-sided polygons?

Number of sides	Number of diagonals
3	
4	
5	
6	
7	

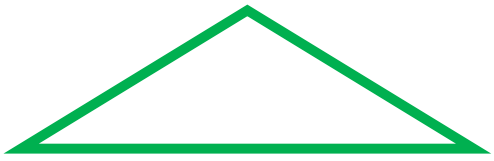
So how do we count diagonals?

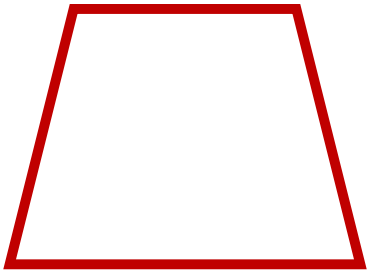


Let's describe a diagonal as a line segment connecting a vertex to another vertex of the shape that isn't a neighboring vertex. Here is a picture describing one diagonal in a hexagon (6-sided polygon). I have labeled the vertices (plural of vertex) with letters. How many diagonals start at A? I drew one and will call it AC.

How many diagonals start at A?
List all of them starting with AC.

Do you agree that a triangle has no diagonals? And that a 4-sided polygon has 2?





Play with 5-sided, 6-sided, and maybe even more-sided polygons

How many diagonals do you find? Can you name them? What patterns can you find? Can you guess and confirm the number of diagonals for polygons with more sides?

My notes on counting diagonals

Did you find any patterns or ways to predict the number of diagonals for any number of sides? Oh how about color - do colors for you go with certain shapes, letters or numbers?

Patterns and ways to predict diagonals

Do colors go with certain shapes for you?

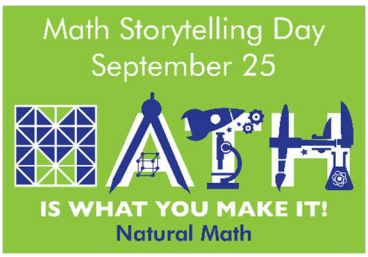
My notes on making triangles

(if we have time) Spaghetti and Triangles!

Use pieces of spaghetti or even cut out lengths of paper to create triangles. Can you make any kind of a triangle? Can any three lengths make a triangle? We will probably pick up with this next time!

Keep on playing at home!

At home keep playing with your spaghetti pieces and see if you can figure out what kinds of pieces can make triangles and other shapes. Can you think of interesting ways to describe shapes like 3-, 4-, and 5- sided figures? Keep your eyes open – where do you see polygons?



I hope you had fun! Remember that September 25 is Math Storytelling Day – can you make up a story about math? See you next time! See you at BugFest?

Have Fun!
Mr. Barman

Notes for parents

Playing with shapes is an interesting way to bridge drawing and numbers. Little did children know that their doodles could be looked at arithmetically!

I was thinking of making the whole hour about counting diagonals but decided to include material on a few other topics. We'll see how the hour goes. We might come back and look at diagonals more carefully next time. You may enjoy looking over the website splashlearn.com/math-vocabulary/geometry/polygon about polygons

Diagonal-counting is a surprisingly rich area to explore. I'm working on a more complete document on this topic available, currently up to 18 pages, via the QR code here or by visiting mathinst.com/diagonals. Feel free to take a look. The result of the approaches investigated there is copied onto the next page.

Thanks for coming to Chapel Hill Math Circle. If you stop by BugFest at the NC Natural Sciences Museum in Raleigh on September 20, 2025, stop by our "Math Neighborhood"!

Till next time!

Dilip

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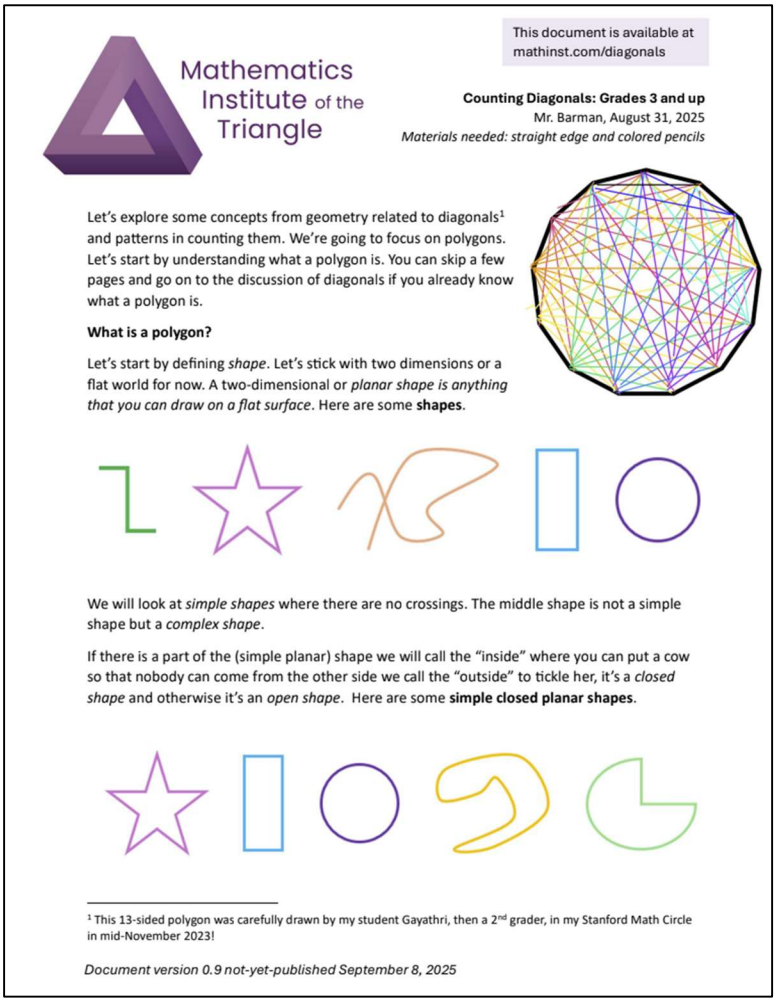

Mathematics Institute of the Triangle

This document is available at mathinst.com/diagonals

Counting Diagonals: Grades 3 and up

Mr. Barman, August 31, 2025

Materials needed: straight edge and colored pencils

Let's explore some concepts from geometry related to diagonals[1] and patterns in counting them. We're going to focus on polygons. Let's start by understanding what a polygon is. You can skip a few pages and go on to the discussion of diagonals if you already know what a polygon is.

What is a polygon?

Let's start by defining *shape*. Let's stick with two dimensions or a flat world for now. A two-dimensional or *planar shape is anything that you can draw on a flat surface*. Here are some **shapes**.

We will look at *simple shapes* where there are no crossings. The middle shape is not a simple shape but a *complex shape*.

If there is a part of the (simple planar) shape we will call the "inside" where you can put a cow so that nobody can come from the other side we call the "outside" to tickle her, it's a *closed shape* and otherwise it's an *open shape*. Here are some **simple closed planar shapes**.

[1] This 13-sided polygon was carefully drawn by my student Gayathri, then a 2nd grader, in my Stanford Math Circle in mid-November 2023!

Document version 0.9 not-yet-published September 8, 2025

Summary of ways presented here to count diagonals D_n in an n-gon
In all these ways, we assume that n is a counting number 3 or larger

The **forward difference / recurrence** way starts with easily finding $D_3 = 0$ and building from there.

$$D_{n+1} = D_n + (n-1);\ D_3 = 0$$

The **unfriendly mayors** (or maybe it should be called *uncommunicative* mayors) method has each of the n vertices connecting not to themselves or their neighbors; we double count so take half.

$$D_n = \frac{n(n-3)}{2}$$

The **n-3 then countdown** method is

$$D_n = (n-3) + (n-3) + (n-4) + (n-5) + (n-6) + \cdots + 1$$

-or-

Add up (n-3) and then all the numbers counting down from (n-3)

We can use the **Triangle Numbers** method as well.

$$D_n = T_{n-2} - 1$$

or

$$D_n = T_n - 2n$$

Finally, we can use combinatorics and binomial coefficients.

$$D_n = \binom{n}{2} - n$$