

Name: _____

Math Circle

Directions: Work on the following problems in your group.

Problem 1. Calculate the expected value of the following experiments:

- (a) Flipping a fair coin, where getting heads is worth 1 point and tails is worth -1 points.
- (b) Rolling a fair 10-sided die, where the outcome is the number rolled on the die.
- (c) Rolling 2 standard 10-sided dice and adding the results together.
- (d) Rolling 2 standard 4-sided dice and taking the highest result.

How would the answer to (c) change if you rolled 100 dice instead?

Problem 2. A weighted coin has a probability of 0.6 of landing on heads. Calculate:

- (a) The probability of landing on tails
- (b) The expected value of a game where this coin is flipped, awarding 10 points if it lands on heads and 5 points if it lands on tails

Problem 3. You and your group are walking around the annual Math Department Carnival when a dastardly figure approaches you with two mysterious options:

- Spin the Wheel-O-Money™
- Get \$10 of cold hard cash

The Wheel-O-Money™ has 15 equally-sized sections on it, 14 of which are labeled *Loser* and 1 of which has a \$100 dollar bill on it. Assuming the wheel is fair, use what you know of expected value to determine if you should spin the Wheel-O-Money™. Explain your answer.

Problem 4. Jane is flipping coins with her friend John. If John's coin lands on heads, Jane gets 5 points from John. If the coin lands on tails, Jane gives John 3 points. Both players start with 20 points every time they begin playing.

(a) Assuming the coin is fair, how many points should Jane expect to win/lose per coin flip?

(b) Do you think this is a fair game, or does one of the players have an advantage?

The first time they play, they flip the coin 10 times. After, they tally up the scores to find out that John has 18 points and Jane has 22.

(c) How many times did the coin land on heads? Tails?

(d) Do you think this is a fair game, or does one of the players have an advantage?

Jane and John are *super* bored, so they play this game 100 times (still 10 flips every time they play the game) and keep track of the results. On average, John gets 26 points and Jane gets 14 points.

(e) Use this information to estimate the probability of flipping and getting heads. How about tails? Is this different than what you assumed?

(f) Do you think this is a fair game, or does one of the players have an advantage?

With the same coin, suppose they now play a modification of this game: rather than 10 flips, they stop playing as soon as someone drops below 0 points.

(g) Using the probabilities estimated in part (e), what is the expected number of flips before someone runs out of points?

Problem 5. What is the expected number of rolls you make on a 6-sided die before rolling:

(a) the number 1:

(b) any even number:

Problem 6. A variation on the Math Circle Battle-pass! We still have the two 6-sided dice (one blue and one white), but now suppose instead the rules are:

(1) You first roll the white die and get a number N

(2) You then roll the blue die until you roll a number less than or equal to N

Your final score (and number of M-Bucks earned) is the sum of the numbers you rolled on the blue die times N . What is the maximum amount of M-Bucks you would pay to play this game?

Problem 7. A building has 10 floors above the basement. If 12 people get into an elevator at the basement, and each chooses a floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop to let out one or more of these 12 people?¹

¹Source: OpenQuant - Ten Floor Building

CHAPEL HILL MATH CIRCLE EXIT TICKET:

August 30, 2025: Expected Value Games

Please remove this sheet, complete it, and return it before the end of our session.

- Did you find today's topic interesting?
- Was the this topic appropriately challenging relative to your background? That is, was the topic neither too elementary nor inaccessibly advanced?
- How could we improve this worksheet for future sessions?
- What did you enjoy about today's topic?
- What did you find particularly challenging?
- Was there anything you thought was too difficult?
- Was there anything you thought was too easy?
- Are there any topics you would be interested in seeing us cover in the future?

