Chapel Hill Math Circle
Session 3 – September 27, 2025: Diagonals part 2
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com
Supplies needed: rulers, spaghetti, paper, colored pencils, optional rope

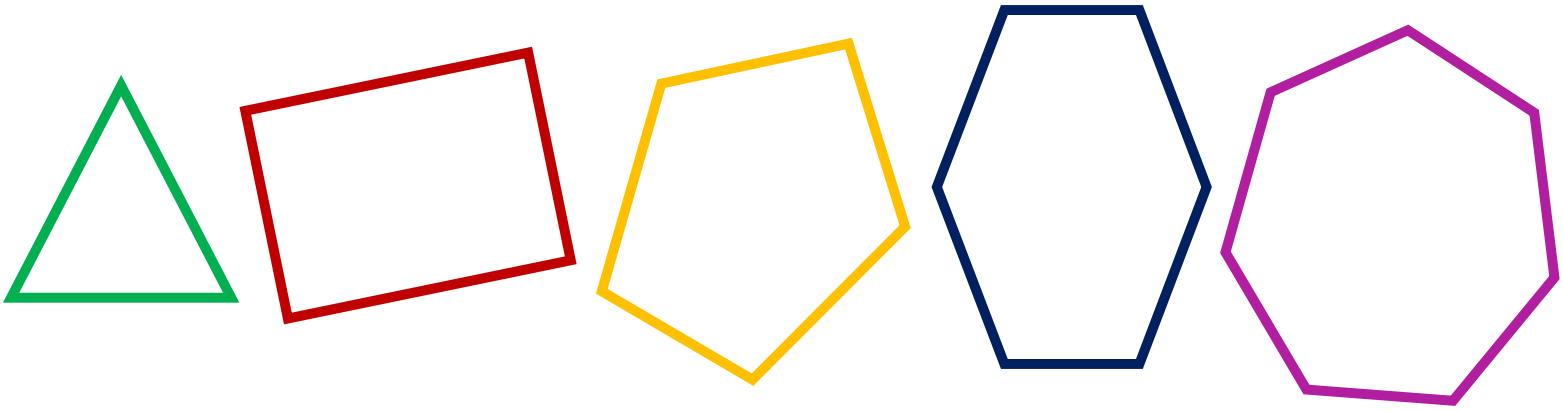
Welcome to Chapel Hill Math Circle! Last time we discussed diagonals in polygons. It's okay if you weren't here; we'll review what we did and go further.

We discussed **closed shapes** where something could be contained. **Polygons** are closed shapes with straight lines. A **simple shape** has no crossing lines. **Regular polygons** have all the lines the same and all of the angles in between the lines the same.

Diagonals

What is a diagonal?

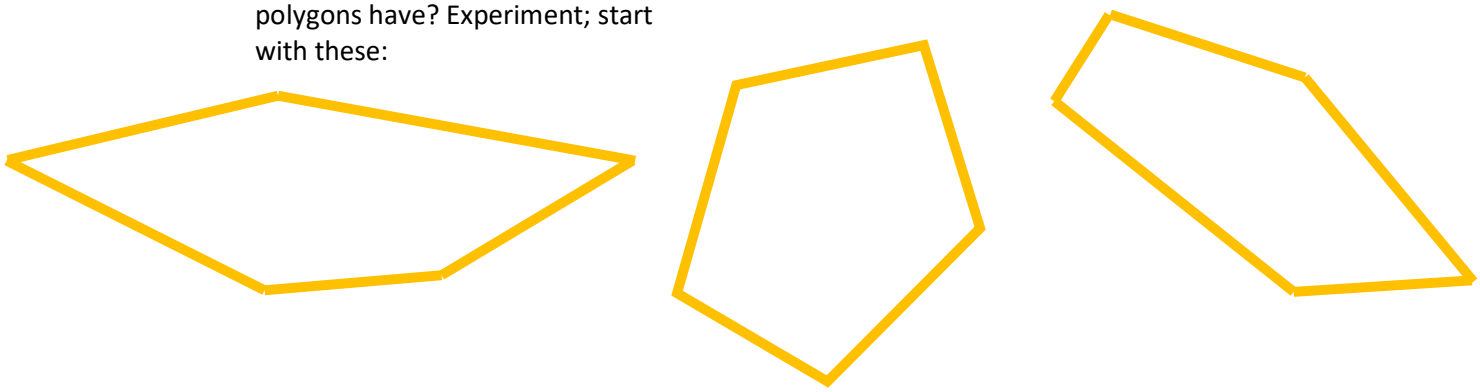
What do you think a diagonal is? If you were here last time, do you remember what we came up with? Can you use different colors to draw all the diagonals in these polygons? Do all triangles have the same number of diagonals? All 4-sided polygons? All 5-sided ones? All any-sided ones?



(Don't peek till I tell you: A diagonal is defined as a line segment between two "vertices" of a polygon that aren't directly connected.)

How many diagonals?

- Did you find that triangles have no diagonals and 4-gons ("quadrilaterals") have 2?
- How many diagonals do 5-sided polygons have? Experiment; start with these:



Make a table of what you have found so far. Do all polygons of the same number of sides have the same number of diagonals? If not then list all the possibilities.

Do you have a prediction of how many diagonals a 6-sided polygon would have? How about 7-sided polygons?

Number of sides	Number of diagonals
3	
4	
5	
6	
7	

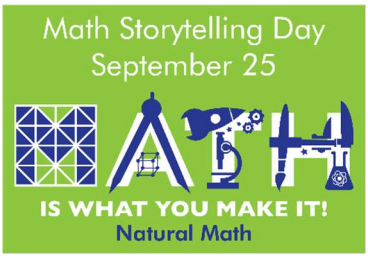
We found last time n-3 then countdown

Let's review the "n-3 then countdown" pattern that we found. Then let's look at how we can find these countdown numbers easily. If we have time, we'll explore a few more ways to count diagonals. Here are two ways that we may explore:

- Forward differences
- Unfriendly mayors

Keep on playing at home!

Can you predict and then, with a drawing or with pieces of spaghetti, make all the diagonals for polygons of 4, 5, 6, and 7 sides? Convince yourself that the number of diagonals is the same whether the polygon is regular (all the sides are the same length and all the angles are the same) or not.



Extend your table – how far can you go? Which method do you like using to count diagonals?

I hope you had fun! See you next time!

Have Fun!
Mr. Barman

Notes for parents

You can see my work-in-progress about diagonals at mathinst.com/diagonals, like I shared last time. Today I'm just leaving a summary of ways that the document leads up to, reproduced below.

Till next time!

Dilip

Dilip Barman dilip@trianglemathinstitute.com

Summary of ways presented here to count diagonals D_n in an n-gon

In all these ways, we assume that n is a counting number 3 or larger

The **forward difference / recurrence** way starts with easily finding $D_3 = 0$ and building from there.

$$D_{n+1} = D_n + (n-1);\ D_3 = 0$$

The **unfriendly mayors** (or maybe it should be called *uncommunicative* mayors) method has each of the n vertices connecting not to themselves or their neighbors; we double count so take half.

$$D_n = \frac{n(n-3)}{2}$$

The **n-3 then countdown** method is

$$D_n = (n-3) + (n-3) + (n-4) + (n-5) + (n-6) + \cdots + 1$$

-or-

Add up (n-3) and then all the numbers counting down from (n-3)

We can use the **Triangle Numbers** method as well.

$$D_n = T_{n-2} - 1$$

or

$$D_n = T_n - 2n$$

Finally, we can use combinatorics and binomial coefficients.

$$D_n = \binom{n}{2} - n$$