Chapel Hill Math Circle
Session 5 – October 25, 2025: Fun with Subtraction
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com
Supplies needed: paper and colored pencils, 3' or slightly longer rope

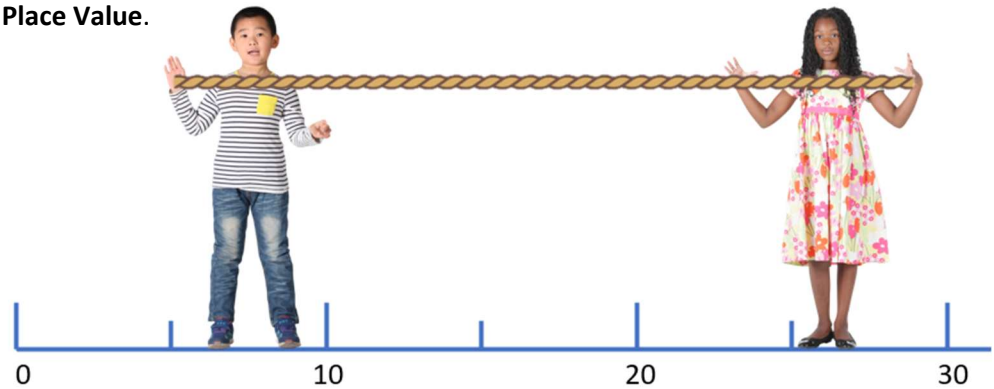
Welcome to Chapel Hill Math Circle! Let's play a bit with subtraction today.

What is subtraction and where can you use it?

Some of us know "subtraction" as "taking away", "minusing", or "difference". Can you come up with some examples from your life where you need to subtract something?

Do you know any rules for subtraction? Do you know any shortcuts?

Let's learn a method I call the **String Method** of subtraction. We will demonstrate this in class. We will also learn **Estimate and Fix, Airport Method, Helicopter Method, One More Day library book Way, Making Change Way, and Place Value**.



Let's try to subtract **62-27**. Pretend that you and a friend are standing on a number line. Your friend is at 62 and you're at 27, and you're holding a string in front of you tightly. If your friend moves to a bigger number, he will drag you to the right, and if you move to a smaller number, you will drag him to the left.

Now, alas, you start crying. "I'm at 27, a difficult number to subtract! Friend, can you move 3 to the right? That will drag me to 27+3 = 30." Your friend moves right to get to 62+3 = 65. So now we can solve an easier problem because the difference is the same! 65-30 is easy; 65-10 is 55, minus another 10 is 45, and minus a third 10 is 35. But 65-30 is the same as our original problem, 62-27! So we have **62-27 = 65-30 = 35**.

There are many other ways to do this subtraction. Here are some ways I've come up with:

- **Estimate and Fix:** 62-27 is about (≈) 60-30, which we can easily see is 30. We'd be exactly correct if we add 2 (we had 62, not 60) and add 3 (we took away 30 but only should have taken away 27) – in other words, 62-27 = 60-30+3+2 = 30+5 = 35.

- **Airport Method:** We are at 27 and want to fly to 62. We get a ride 3 miles to the airport and then fly from the airport at 30 to the airport at 60. We get a ride 2 more miles to our destination. So 62-27 = go from 27 to 62 = 3 (to get to 30) + 30 (fly to 60) + 2 (to get to 62); 62-27 = 3+30+2 = 35.

- **Helicopter Method:** We still go to the airport, but now we ride a helicopter that takes us to the destination – complete with a helipad! So 62-27 entails taking a ride 3 miles from 27 to 30 and then flying 32 miles from 30 to 62; 62-27 = 3+32 = 35.

- **One More Day library book Way:** You have 62 books and the library requests that you return 27 of them. You are happy to return 22 now (because 62-22 is easy) and ask if you can hang on to 5 of them for another day (or two). You return 22 now and 5 later to return 27. So 62-27 = 62-22-5 = 40-5 = 35.

- **Making Change Way:** Let's pretend that we are paying for something. We have $62 and are buying something for $27. We pay $30 with three $10 bills or a $20 bill and a $10 bill. The store gives us back $3 in change as we overpaid. So 62-27 = 62-30+3 = 32+3 = 35.

- **Place Value:** This is the traditional way to subtract. 93-21? 9 tens minus 2 tens is 7 tens. 3 ones minus 1 one is 2 ones. So we have 7 tens and 2 ones or 72. But we have to break or borrow for our problem of 62-27. We see that we can't take 7 ones from 2 ones but we can break 6 tens into 5 tens and 10 ones giving us 5 tens and 12 ones minus 2 tens and 7 ones or 3 tens and 5 ones, 35. I like place value when the problem is easy and there is no borrowing.

You may know or will learn in school borrowing with place value. I like the other ways I'm sharing here generally better; they are faster and can often be easily done in your head with no paper. When you have problems where simple place value works best, use that – for example, 67-42 is just 25; 6 tens minus 4 tens = 2 tens and 7 ones minus 2 ones is 5 ones for 25.

Take some time and try the string method. Then try the problems on the next page. Use all of these ways if you can. For each problem note which way you found was the easiest with a star and jot down any notes. Work with a partner; each pair of students should have a rope or string that is about 3' or 4' wide.

Are you not comfortable with subtraction? No problem! You can use the time to play with "taking away". Grab some pencils and look for patterns when you take some away. Start with 3 pencils – can you take 1 away? Or 2? Or 3? What are you left with in each case? If you have 15 and take away 10, what do you get? How about if you have 16 and take away 10? Experiment!

For each subtraction problem, which way do you like best to solve it? Write the answer and any notes. Take a quick peek at my work on the next page to get an idea of what your table might look like. Everybody's will likely be different – we all have different preferences.

	String	Estimate and Fix	Airport	Helicopter	One More Day Library	Making Change	Place Value
72-37							
98-28							
92-28							
46-27							
86-48							
98-12							
85-28							
126-32							
183-68							
158-72							
193-177							
125-86							

	String	Estimate and Fix	Airport	Helicopter	One More Day Library	Making Change	Place Value
72-37	*75-40	*72-40+3	3+30+2	3+32	72-32-5	(same as E+F)	6 13 - 3 7
98-28 70							*70
92-28 64	*94-30	92-30+2	2+60+2	*2+62	92-22-6	(E+F)	8 12 - 2 8
46-27 19	49-30	46-30+3	3+10+6	3+16	*46-26-1	(E+F)	3 16 - 3 7
86-48 38	88-50	90-50-4+2	2+30+6	*2+36	*86-46-2	86-50+2	7 16 - 4 8
98-12 86							*86
85-28 57	*87-30	85-30+2	2+50+5	*2+55	85-25-3	(E+F)	
126-32 94	124-30	130-30-4-2	*8+90-4	8+86		126-40+8	*94
183-68 115	185-70	180-70+3+2	2+110+3	2+113		*183-70+2	
158-72 86	156-70	*160-70-2-2	8+80-2	8+78		158-80+8	*86
193-177 16	96-80	200-180-7+3	3+10+3	*3+13		193-180+3	
125-86 39	129-90 = 139-100	125-90+4	4+30+5	4+35	*125-85-1	(E+F)	

In some cases I picked more than one way that I thought was my favorite. Note that Making Change is often the same as Estimate and Fix. Also note that in Airport Method you sometimes fly past where you are going and drive back, like for 126-32; the closest “airport” (multiple of 10) to 126 is 130 and not 120. Sometimes when one way is by far the easiest, like Place Value for 98-12, I skipped the other ways. I usually don’t like Place Value; I showed several “not fun” examples and then I stopped.

Subtracting 3s and 5s from 20 – what can you get?

Let's conclude with playing with subtraction. What can I get[1] starting at 10 if I can subtract any number of 4s and 7s? I can get:

- 10 - 7 = 3
- 10 - 4 = 6
- 10 – 4 – 4 = 2
- 7 – 4 = 3 (again)

And that's it. I can get 2, 3, and 6.

Please try this problem: what can I get, starting at 20, and subtracting any number of 3s and 5s? Take a few minutes to try this.

I used a "brute force" method for the problem above. Can you think of any "elegant" ways?

[1] If you know about negative numbers, we are looking for only non-negative ones: 0, 1, 2, 3, ….

Inspired by my 2nd grade student Anika and a problem we sometimes look at in math circle, the Frobenius Coin Problem, here is one way to solve it. Let's start at 20 and count backwards. I'm going to count in rows of 5. Why 5? Because if I can make a number, say 12 (3+3+3+3), I can make everything under it by subtracting more 5s. (I could have made a grid that was 3 wide as well – do you agree?)

- Subtracting 5s, I can get to 15. That means that I can also get to 10 and 5.
- Okay, now let's subtract 3 from 20. I get to 17 – and can definitely get to 12, 7, and 2 by subtracting 5s.
- Let's continue subtracting 3s and get to 14 (and 9 and 4 under it), 11 (and 6 and 1), and 8 (and 3)

What's left are the numbers I can't make – 20, 19, 18, 16, and 13.

Another of my students, 2nd grader Luc, used a similar method in October 2025 that he called a grid. My 1st grade student Reyansh, also in October 2025, used a "tree"; he subtracted 3s to the left and 5s to the right. I am lucky to get such smart and creative students – like you!

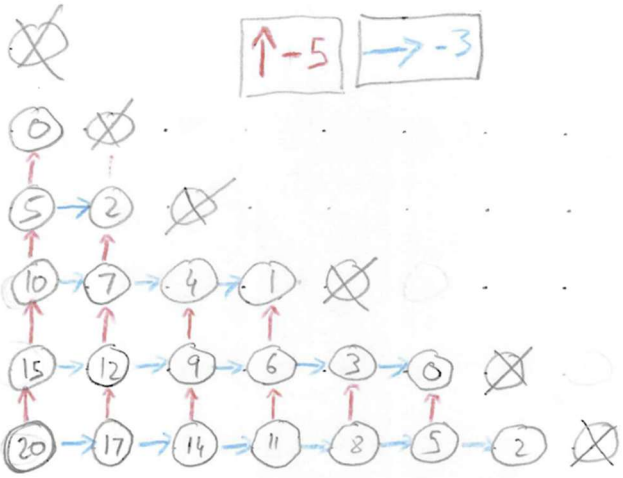
I hope you had fun. See you next time!

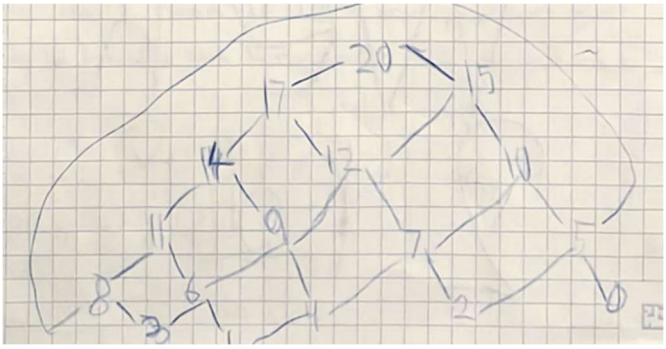
20	19	18	17	16
15	14	13	12	11
10	9	8	7	6
5	4	3	2	1

20	19	18	17	16
15	14	13	12	11
10	9	8	7	6
5	4	3	2	1

20	19	18	17	16
15	14	13	12	11
10	9	8	7	6
5	4	3	2	1

20	19	18	17	16
15	14	13	12	11
10	9	8	7	6
5	4	3	2	1





Have Fun!
Mr. Barman

Notes for parents

Subtraction can be daunting. In years of teaching 2nd grade I've come up with a number of ways to make subtraction easier and hope that these ways will help your child become more comfortable with this important operation. By the way, don't tell your child that in 5th or 6th grade they'll probably learn that they never have to subtract again – they can simply add a number's negation (5 - 3 = 5 + (-3))!

Each of the techniques has its merits:

- I love the ability to enact the **String Method**.
- In my classes I emphasize the importance of estimating; sometimes it's more important to be fast and close rather than to take a long time and be exact. This is where **Estimate and Fix** is helpful – sometimes we can speed it up if we don't need the Fix part.
- The **Airport Method** is a very nice one to graphically introduce on the number line and visualizes subtraction as addition; the **Helicopter Method** is similar.
- The **One More Day library book Way** is a version of Estimate and Fix and, for problems where the units digit of the subtrahend is bigger than that of the minuend[2], it can be helpful.
- The **Making Change Way** may be more interesting to parents than the current generation of children. In an age of moving toward a cashless society, many children are not familiar with coins. I do encourage parents to integrate some cash into their shopping, perhaps giving some responsibility to their 1st through 4th grade children to budget, use, and estimate that cash perhaps in buying produce or some other subset of the family's shopping. In any case, for those comfortable with making change, this technique can be handy.
- I generally avoid **Place Value** to subtract, except for easy cases, as it is usually more difficult to use for quick mental math. It clouds the underlying numeracy and can become a mindless algorithm, making users immune to common sense errors.

All promote mental math and not using a calculator where one can easily throw out common sense and make obvious errors.

I know that our math circle includes 1st graders and some of them may have much more rudimentary knowledge of subtraction. I will try to manage that in class and give them easier concepts to consider that may help lay the groundwork for subtraction-readiness. For example viscerally understanding "take away" by using objects can both reinforce the definition and implications, as well as plant the seeds to show why addition and subtraction are inverse operations.

I don't know if we'll get to the repeated subtraction. I was so impressed with my students who came up with elegant solutions for it. Till next time!

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[2] In a-b=c, a is the "minuend", b is the "subtrahend", and c is the "difference".