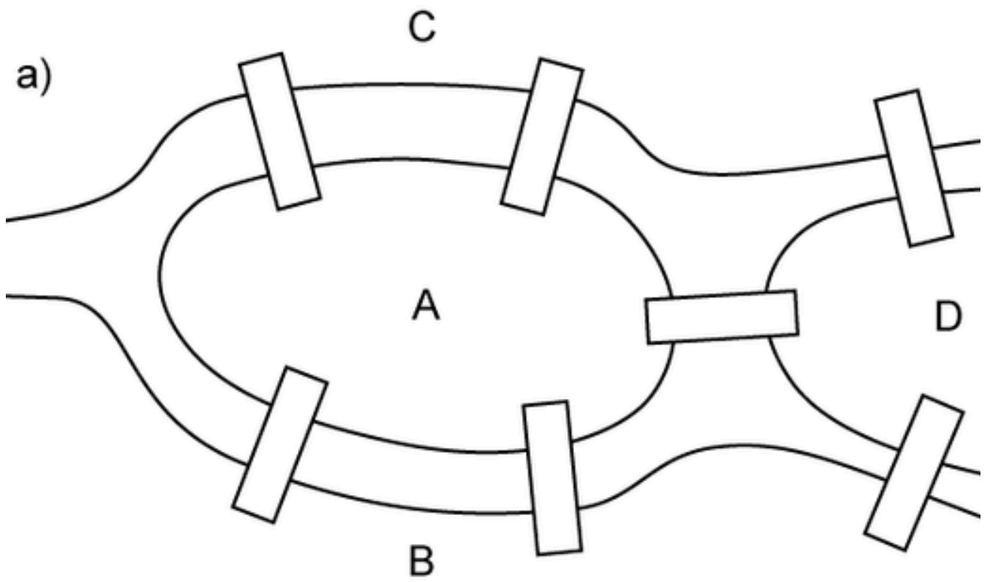
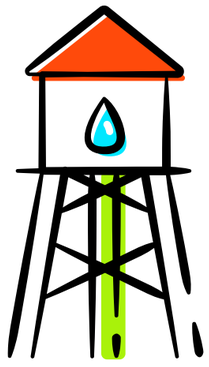
The Bridges of Königsberg

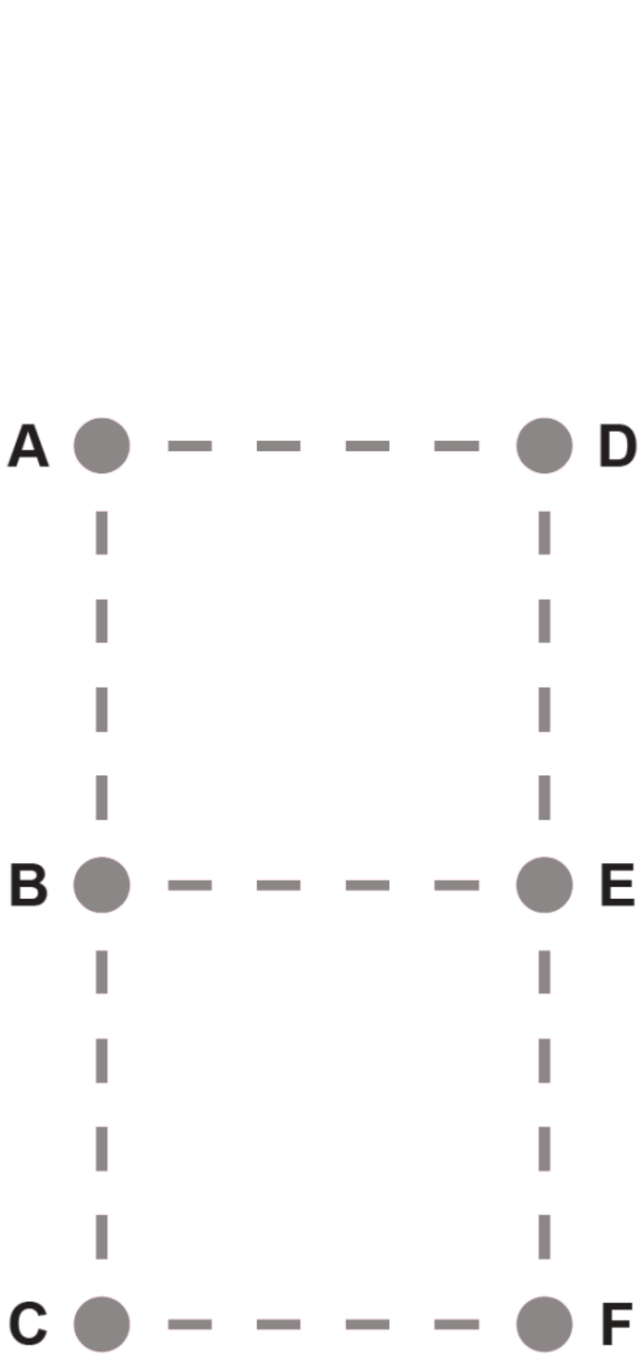
Chapel Hill Math Circle

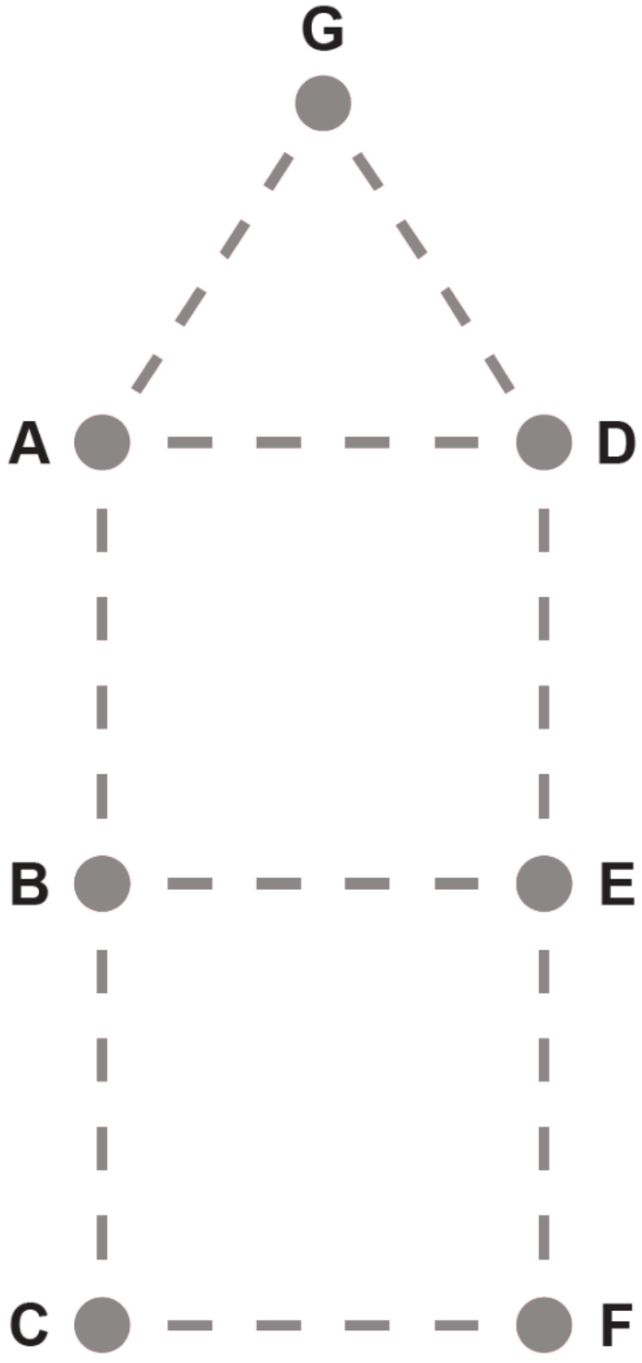


Last week, we turned our maps into graphs. What would this problem look like if we turned it into a graph? What would be the vertices/nodes? What would be the edges?

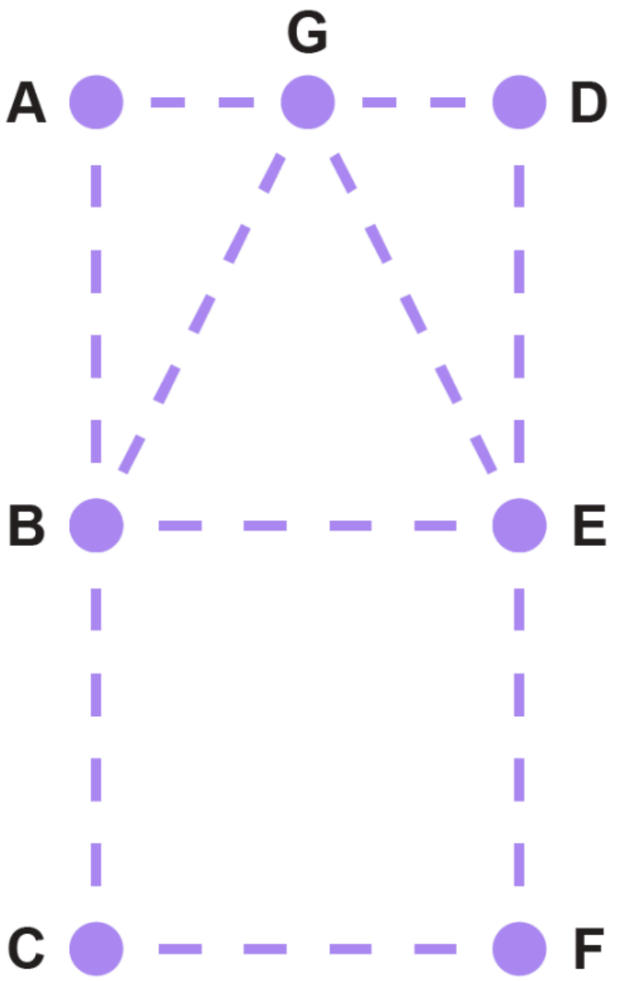
Suppose you have a neighborhood with only three houses. Each house needs to be connected to a set of three utilities (gas, water, and electricity). Your challenge is to connect all three houses to each of the utilities. Below is a diagram for you to draw out where each utility line/pipe should go. The catch is that the lines you draw to connect each house to a utility can't cross. Also, you can't draw lines through another house or utility plant. In other words, you can't draw House 1's water line through House 2 or through the electricity plant. (Courtesy of University of Waterloo Math Circle).







How can we tell if a graph problem is possible or impossible?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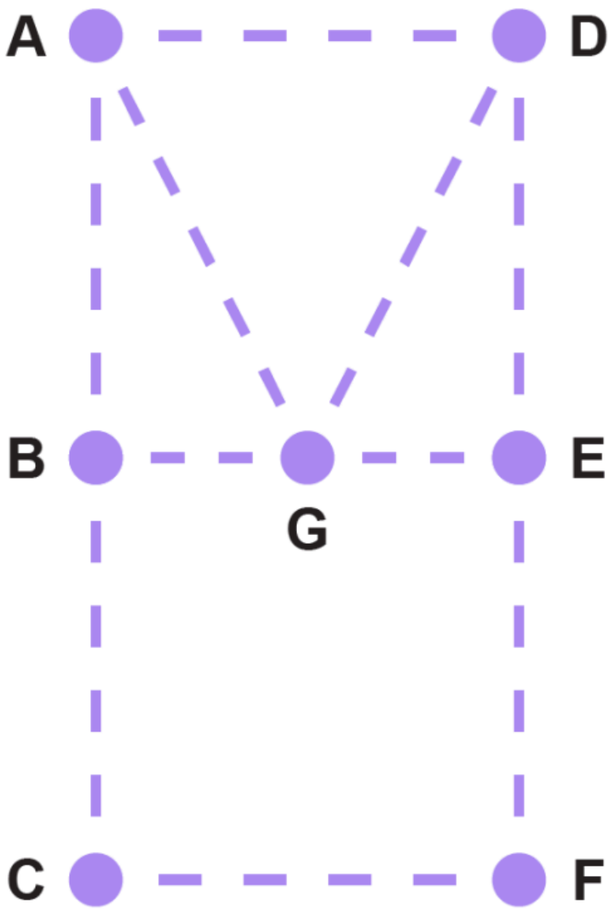
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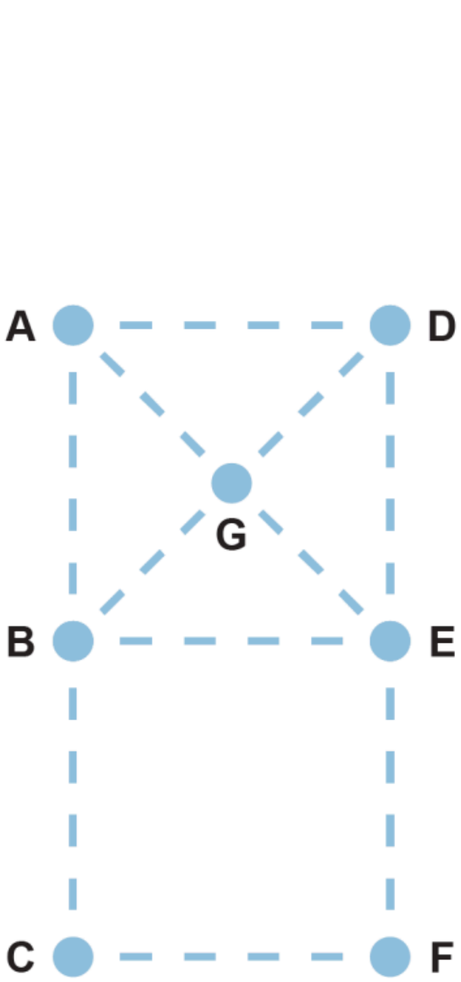
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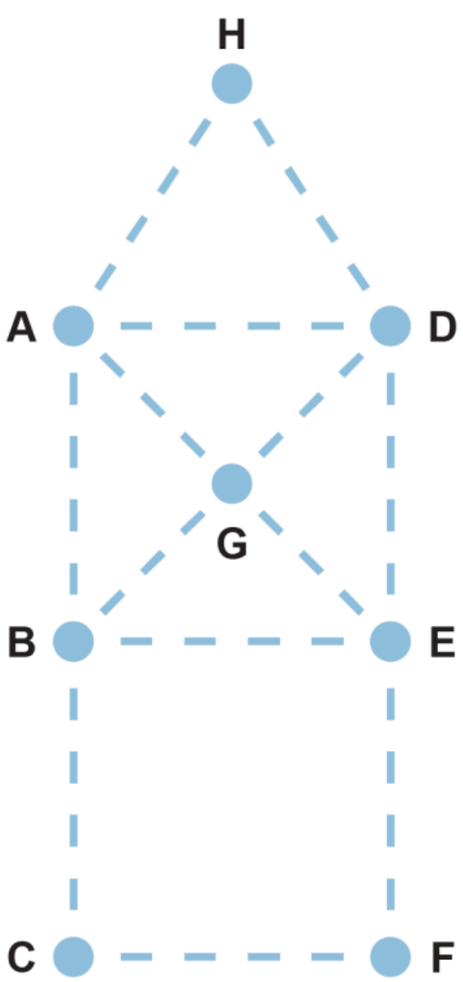
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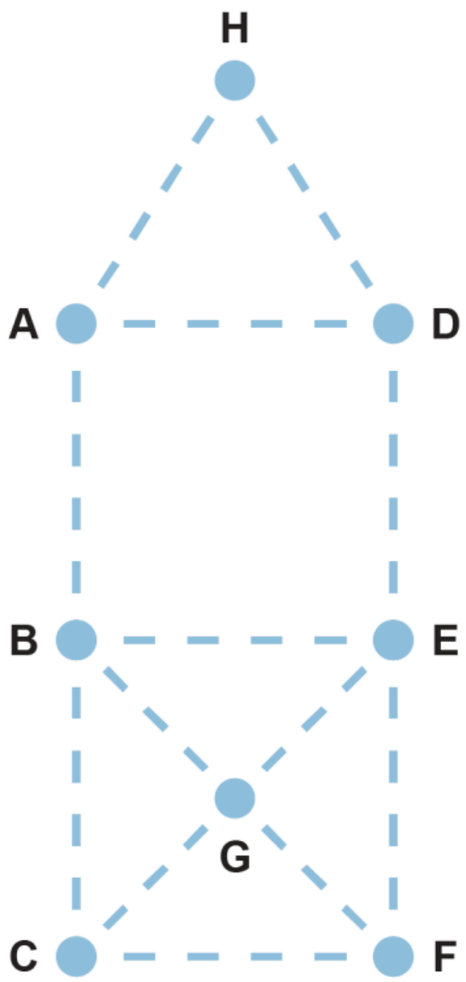
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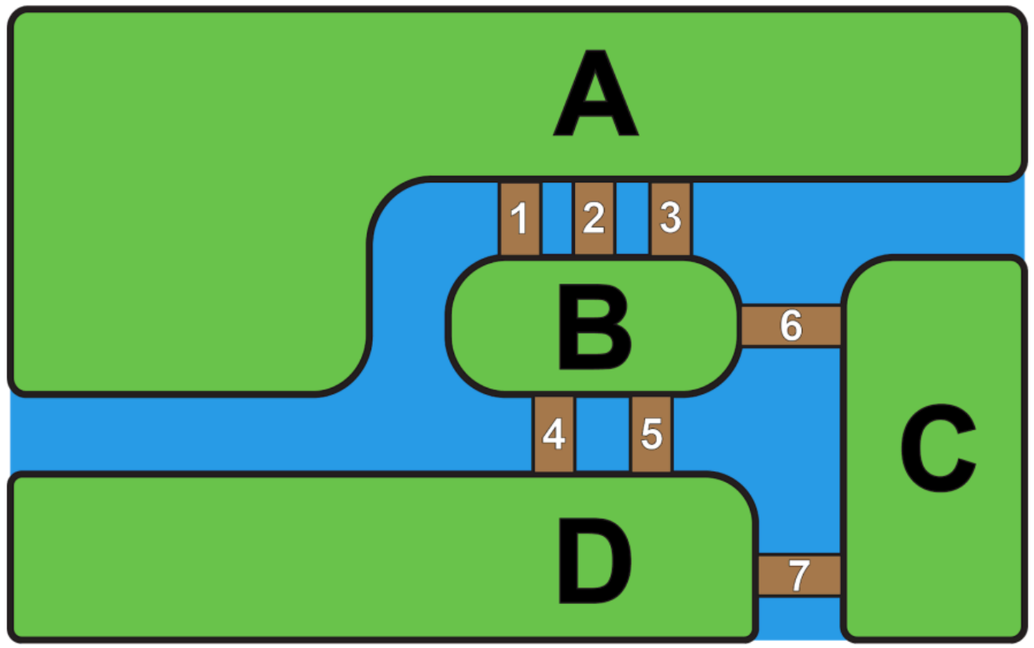
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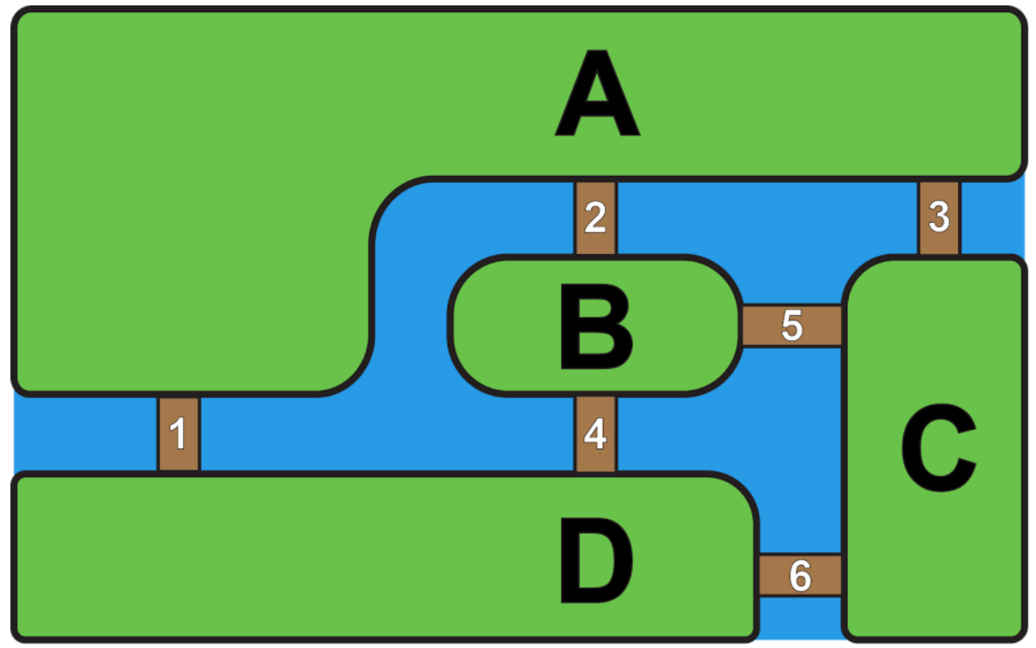
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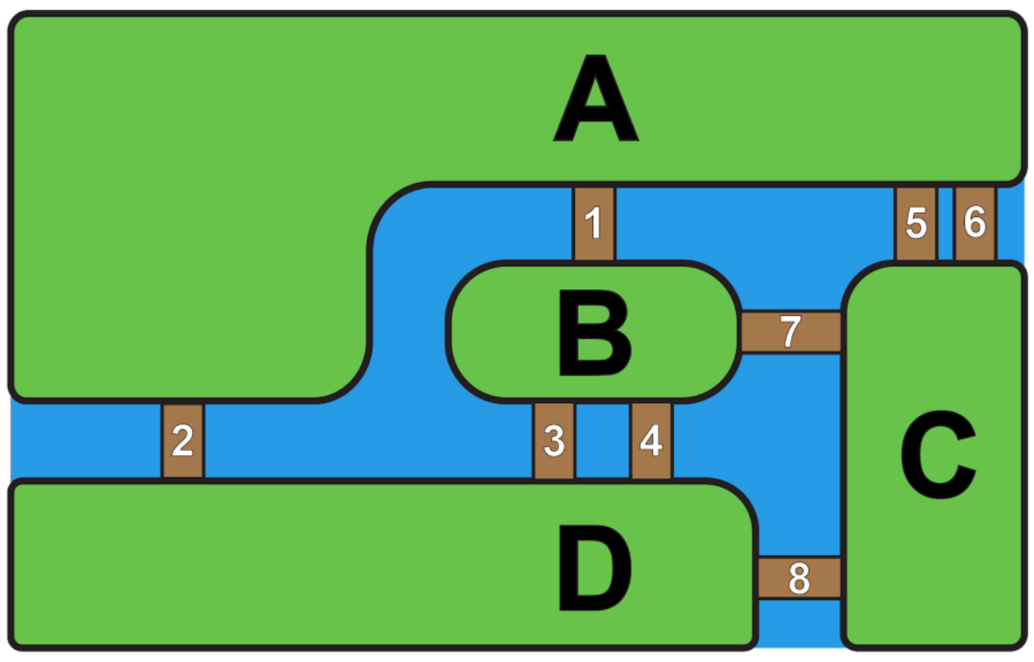
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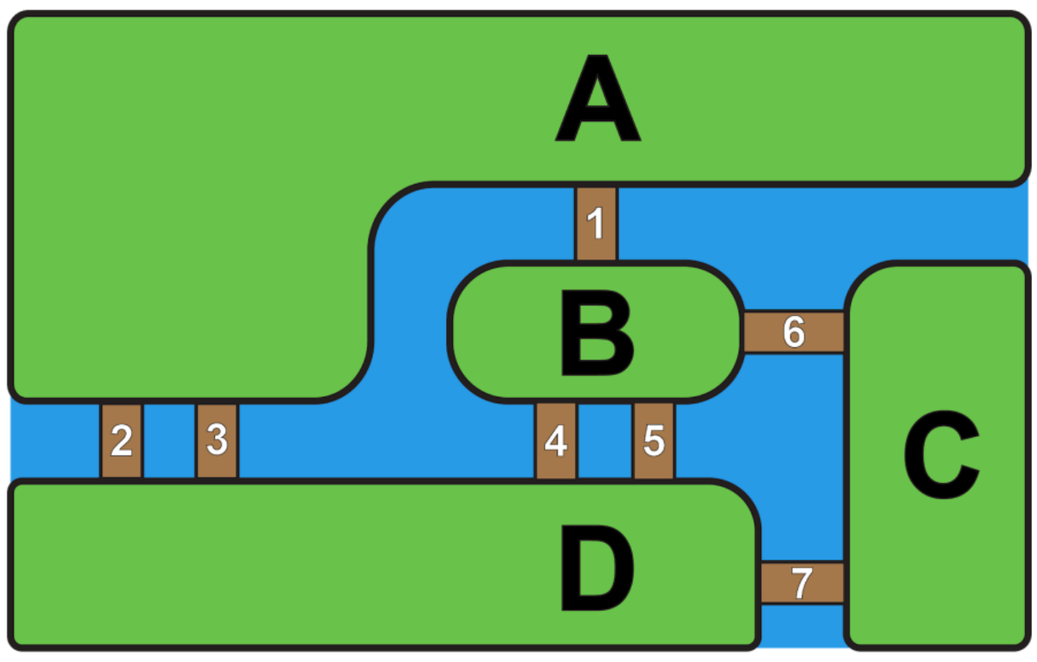
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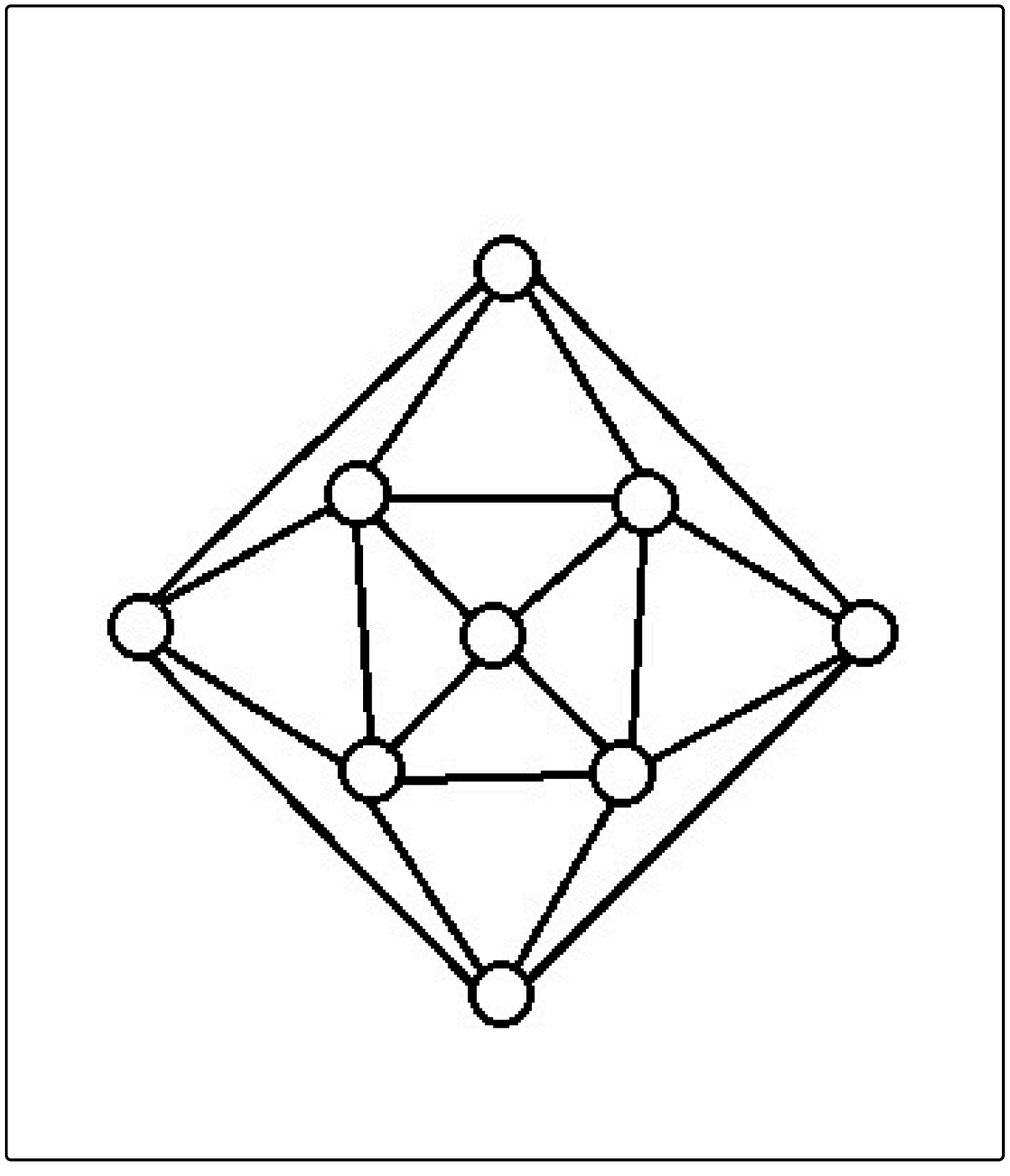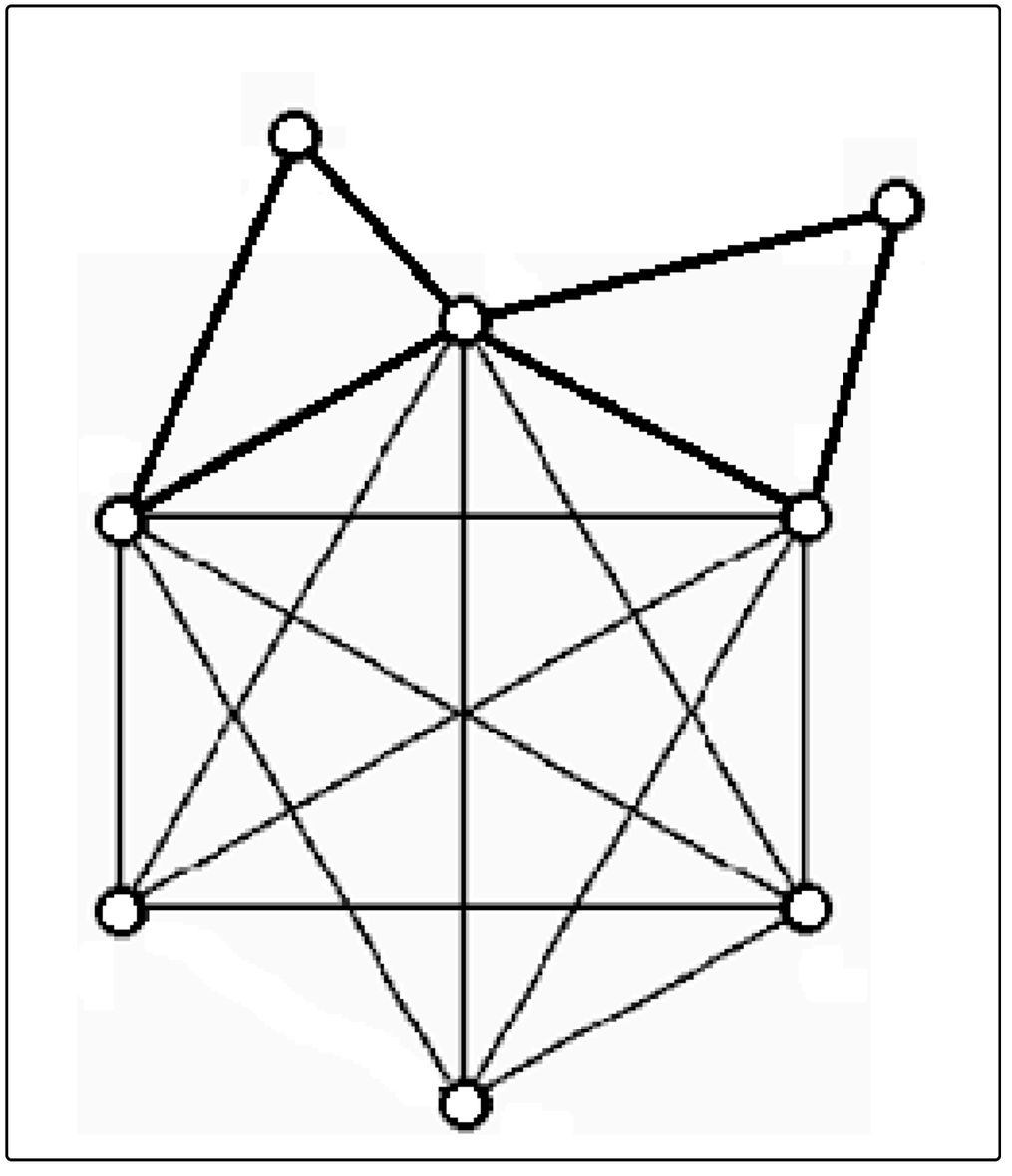
Remember these graphs from last session? Can you travel along them crossing every edge exactly once?